REAL NUMBERS

IMPORTANT FORMULAE AND DEFINITIONS

GENERAL

The study of real numbers is very important for every competitive examination. About 25% of the questions are usually set on the properties of real numbers, simplification of expressions involving numbers and arithmetic operations, finding powers or extracting roots of simple, fractional or decimal numbers, use of algebraic formulae and geometrical results in the evaluation of simple expressions, etc. Given below are some of the important and basic definitions and concepts which are very useful in solving such questions:

Abstract Number. A number not attached to any particular thing is called an abstract number, e.g., one, four, five, etc.

Concrete Number. A number of particular units such as three girls, seven pens, etc. is called a concrete number.

Digits. All numbers are written by means of symbols, 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, which are called digits.

Rational Number. A real number expressible in the form $\frac{p}{q}$, where p and q are integers and q is not equal to 0, is called a rational number. All positive and negative integers, fractions, finite decimal numbers and infinite recurring decimals are called rational numbers.

Irrational Numbers. Real numbers not expressible in the form $\frac{p}{q}$, where p and q are integers and q is not equal to 0, are called irrational number, e.g., $\sqrt{2}$, $\sqrt{3}$ π, etc.

Perfect Number. A number that is equal to sum of all its divisors except itself, is called a perfect numbers, e.g., 6 = 1 + 2 + 3.

Prime Numbers. A whole number greater than one having only two factors ---- unity and itself, is called a prime number, e.g., 2, 3, 5, 7, 11, 13, 17, etc.

Even and Odd Numbers. A whole number divisible by 2 or ending in 0, 2, 4, 6 or 8 is called an even number and the numbers ending in 1, 3, 5, 7 and 9 which are not divisible by 2 are called odd numbers.

Twin Primes. Prime which differ by two are called twin primes, e.g., (11, 13) and (17, 19) are examples of twin primes.

Composite Numbers. The numbers other than prime numbers are called composite numbers.

Goldback's Statement. Every even number greater than 4 is the sum of two prime numbers.

Absolute Value of a Real Number. A real number x is said to have an absolute value $|x|$ which is defined as:

$$|x| = x \text{ if } x \geq 0$$

and $$|x| = -x \text{ if } x < 0$$

Thus $|x|$ is always positive and is equal to numerical value of x.

Consecutive Numbers. A series of numbers in which each number is greater

by 1 than the number which precedes it is called consecutive numbers; e.g., 3, 4, 5, 6, 7.

Divisibility Test for Whole Numbers

(a) A number is divisible by 2 if the unit place digit in it is an even number.

(b) A number is divisible by 3 if the sum of its digit is multiple of 3.

(c) A number is divisible by 4 if the number formed by last two digits on the right side of the number is divisible by 4.

(d) A umber is divisible by 5 if it ends in 0.or 5.

(e) A number is divisible by 6 if it is an even number and the sum of the digits is divisible by 3.

(f) A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

(g) A number is divisible by 9 if the sum of its digits is divisible by 9.

(h) A number is divisible by 10 if it ends in zero.

(i) A number is divisible by 11 if the difference of the sums of alternate digits is zero or multiple of 11.

(j) A number is divisible by 12 if it is divisible by 3 as well as 4.

(k) The real number $10^n - 1$ is always divisible by 9, where n is a natural number. It is divisible by 11 if n is even.

(l) The number $(N^n - N)$ is always divisible by n for all values of N except zero. For example $9^7 - 9$ is divisible by 7... etc.

Squares of Numbers. (1) The squares of natural numbers never end in 2, 3, 7 and 8. (2) A number ending with odd number of zeros can never be a perfect square. It is necessary that the number of zeros in the end should be even. (3) A number ending with 5 as a unit digit can be a perfect square if the last two digits of the number are 25 and after leaving 25, the remaining number is a product of two consecutive numbers, e.g., 13225 is perfect square as the last digits are 25 and 132 is product of 11 and 12.

PRACTICE PROBLEMS

1. How many times does the digit three occur between 100 and 200?

(a) 10 (b) 11
(c) 19 (d) 20

2. The square root of 0.9 is equal to

(a) 0.03 (b) + 0.3
(c) 0.94 (d) + 0.94

3. Which of the following fractional numbers is the smallest?

(a) $\frac{2}{11}$ (b) $\frac{3}{11}$
(c) $\frac{2}{13}$ (d) $\frac{4}{15}$

4. The fractions $\frac{3}{7}, \frac{4}{9}, \frac{6}{7}$ and $\frac{7}{9}$ when arranged in the increasing order will appear as

(a) $\frac{3}{7}, \frac{4}{9}, \frac{6}{11}, \frac{7}{9}$ (b) $\frac{3}{7}, \frac{6}{11}, \frac{4}{9}, \frac{7}{9}$
(c) $\frac{3}{7}, \frac{4}{9}, \frac{7}{9}, \frac{6}{11}$ (d) $\frac{6}{11}, \frac{7}{9}, \frac{4}{9}, \frac{3}{9}$

5. The value of the expression 1011^2 is equal to

(a) 1022121 (b) 1000341
(c) 1021121 (d) 1012221

6. If $\frac{434}{7} = 62$, $\frac{0.000434}{0.14} = ?$

(a) 0.31 (b) 0.031
(c) 0.0031 (d) 3.1

7. When a number is divided by 125, the remainder is 82. When the same number is divided by 25, the remainder will be

(a) 8 (b) 9
(c) 6 (d) 7

8. If 185 x 28 = 5180, 51.8 ÷ 18.5 =

(a) 0.28 (b) 2.8
(c) 28 (d) 280

9. Rashid engaged a labourer for 30 days on the condition that he will be paid at the rate of Rs. 50 per day for the days he works, and he will be fined Rs. 10 per day for the days he is absent. If he receives Rs. 1200 after 30 days, the number of days he was absent is
(a) 6 (b) 5
(c) 8 (d) 4

10. If seven numbers, each divisible by 4 are added, the sum will be divisible by
(a) 7 (b) 4
(c) 28 (d) 9

11. In an examination there were 25 questions. 2 marks were allotted for each correct answer and ½ mark was deducted for each wrong answer. A student who answered all the 25 questions got 37 ½ marks. How many questions did he answer wrong?
(a) 5 (b) 10
(c) 8 (d) 12

12. $\frac{1}{2}$ of 444 ÷ 2.22 = ?
(a) 222 (b) 492.84
(c) 100 (d) 22

13. 2244 ÷ 0.88 = ? × 1122
(a) 20.02 (b) 20.2
(c) 19.3 (d) 2.27

14. $x\sqrt{0.09} = 3; x = ?$
(a) 10 (b) $\frac{1}{3}$
(c) $\frac{1}{10}$ (d) 1

15. 25% of 4 ÷ 4% of 25=?
(a) 1 (b) 3
(c) 0 (d) 67

16. If x % of 60 = 48, then x = ?
(a) 80 (b) 60
(c) 90 (d) 40

17. $1\frac{3}{4} - 1\frac{3}{5} = ?$
(a) .16 (b) .2
(c) .15 (d) $\frac{13}{20}$

18. Subtract $27\frac{5}{14}$ from $43\frac{1}{21}$
(a) $15\frac{17}{21}$ (b) $15\frac{8}{21}$
(c) $15\frac{16}{21}$ (d) $15\frac{4}{21}$

19. If $a > b$ and $b > c$, then :
(a) a = c (b) a > c
(c) c > a (d) a < c

20. $\frac{3}{7} + ? = 3\frac{6}{7}$
(a) $4\frac{2}{7}$ (b) $5\frac{2}{7}$
(c) $3\frac{4}{7}$ (d) $3\frac{3}{7}$

21. $\sqrt{\frac{?}{12}} = 6$
(a) 5184 (b) 72
(c) 2 (d) 12

22. 12% of 1200 = ?
(a) 124 (b) 120
(c) 140 (d) 144

23. $\frac{4}{5} - \frac{11}{15} = ?$
(a) $\frac{11}{15}$ (b) $\frac{1}{15}$
(c) $\frac{44}{75}$ (d) $\frac{15}{20}$

24. 4.0 + ? = 4.02 + 2.4
(a) 2.06 (b) 2.42
(c) 6.42 (d) 2.6

25. If $7\frac{1}{7}$ % of X = 40, then X =
(a) 505 (b) 560
(c) 605 (d) 506

26. If X% of 60=48, then X=
(a) 90 (b) 80
(c) 70 (d) 30

27. By how much is $\frac{3}{7}$ larger than 20 % of 2?
(a) $\frac{1}{35}$ (b) $\frac{1}{7}$
(c) $\frac{1}{4}$ (d) $\frac{4}{7}$

28. 20% of 110 + 10% of 220 = ?
(a) 30 (b) 42
(c) 44 (d) 22

29. If 2x + 3y = 8 and y = 2x then what is the value of z.
(a) -1 (b) 2
(c) 0 (d) 1

30. $\frac{1}{9}$th of 90.90 = ?
(a) 10.1 (b) 1.01
(c) 11.1 (d) 10.01

31. 2.4% of 400 =?
(a) 9.1 (b) 9.3
(c) 9.6 (d) 9.8

32. (4% of 400) (3% of 300)=?
(a) 16 (b) 144
(c) 12 (d) 48

33. $4 \times 9 \div \sqrt{144} = ?$
(a) 3 (b) $2\frac{5}{7}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

34. Which of the following is greater than $\frac{1}{4}$
(a) $\sqrt{\frac{1}{4}}$ (b) $\frac{1}{25}$
(c) 0.04 (d) $\left(\frac{1}{4}\right)$

35. Which of the following has the greatest value?
(a) 0.3 (b) $\frac{1}{3}$
(c) $\frac{2}{5}$ (d) $\sqrt{0.3}$

36. Which pair adds up to a number greater than 5?
(a) $\frac{13}{5}, \frac{11}{6}$
(b) $\frac{11}{4}, \frac{8}{3}$
(c) $\frac{7}{3}, \frac{11}{5}$
(d) None of these

37. 253 ÷ 23 of 11 =?
(a) 0 (b) 1
(c) 18 (d) 11

38. 0.1% of 10 =
(a) 1 (b) 0.1
(c) 0.001 (d) 0.001

39. 73% of 1.9 =?
(a) 1.377 (b) 1.267
(c) 1.387 (d) 1.287

40. $\frac{3}{4}$ of 68 is less than $\frac{2}{3}$ of 114 by
(a) 12 (b) 25
(c) 35 (d) 48

TIME, DISTANCE AND WORK

IMPORTANT FORMULAE AND DEFINITIONS

TIME AND DISTANCE

1. While solving the problems on time and distance the following rules should always be observed.

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Time taken to cover the above distance}}$$

Total Distance Travelled = Speed x Time taken

$$\text{Time taken} = \frac{\text{Total distance}}{\text{Average speed}}$$

2. When a train crosses a pole or a man standing on a platform/near the line, it has to cover its own length with the given speed.

3. When a train passes a railway platform, a bridge or a tunnel, it has to cover the length of the bridge, the platform or the tunnel in addition to its own length.

4. When a train is passing another train completely (moving in same or opposite direction) it has to cover a distance equal to the sum of the lengths of the two trains.

5. If two bodies are moving in the same direction with speeds u and v km/hr starting from the same point, their relative speed is (u–v) km/hr and it is (u + v) km/hr when they are moving in opposite directions.

6. If a man can row at the rate of u km/hr in still water and if v km/hr is the speed of the river, then (u + v) km/hr is the speed of the boat down the stream and (u – v) km/hr is the speed upstream.

TIME AND WORK

If A can do a work in 8 days, it means he can do $\frac{1}{8}$th of the work in a day. Similarly, if many persons work together, their work for one day will be the sum of the works they can separately do in a day.

PRACTICE PROBLEM

1. **10 men can complete a job in 14 days. How long will it take 4 men to finish the same job if they work at the same rate?**
   (a) 33 days (b) 35 days
   (c) 37 days (d) 39 days

2. **15 men can complete a job in 10 days. How long will it take 8 men to finish the same job if they work at the same rate?**
   (a) $14\frac{3}{4}$ days (b) $16\frac{3}{4}$ days
   (c) $18\frac{3}{4}$ days (d) $20\frac{3}{4}$ days

3. **A rectangular room is 6 m long, 5 m wide and 4 m high. The total volume of the room in cubic metres is**
   (a) 24 (b) 30
   (c) 120 (d) 240

4. **A can do a piece of work in 10 days and B can do it in 15 days. The number of days required by them to finish it, working together is**
   (a) 8 (b) 7
   (c) 6 (d) 4

5. **Zaheer can do a wok in 80 hours. If he and his son work together, the time taken is 20 hours. The son working in the same capacity as**

when he was working with his father, can finish the work in?
(a) 20 hours (b) 25 hours
(c) 50 hours (d) 60 hours

6. If 4 men or 7 boys can do a work in 29 days, then 12 men and 8 boys will do the same work in:
(a) 9 days (b) 8 days
(c) 7 days (d) 10 days

7. 15 men can do a piece of work in 20 days, in how many days can 25 men finish the same work?
(a) 12 (b) 15
(c) 2 (d) 20

8. If 1200 men in a fort have provisions for 28 days. After 4 days, 300 men leave the fort. How long will food last now?
(a) 30 days (b) 32 days
(c) 34 days (d) 42 days

9. A cistern normally filled in 8 hours, takes 2 hours longer due to a leak. If the cistern is full the leak shall empty it in.
(a) 24 hrs (b) 30 hrs
(c) 20 hrs (d) 40 hrs

10. A train running between two stations arrives at its destination 10 minutes late when it travels at 40 km/hr, and 16 minutes late when it travels at 30 km/hr. The distance between the two stations is
(a) 10 km. (b) 15 km.
(c) 12 km. (d) 24 km.

11. A pipe can fill a tank in 15 hours. The tank develops a hole and 10% of water leaks out. The pipe will now fill the tank in
(a) 16 hrs 40 minutes
(b) 18 hrs 40 minutes
(c) 20 hrs
(d) 17 hrs 30 minutes

12. A clerk walks from his house at 4 km/hr and reaches his office 5 minutes late. If his speed is 5 km/hr, he will reach his office 10 minutes early. How far is his office from home?
(a) 5 km. (b) $\frac{5}{12}$ km.
(c) $\frac{5}{2}$ km (d) 15 km.

13. A group of 10 students working an hour per day complete a work in 12 days. 12 students in the group working one hour per day will be able to finish it in
(a) 8 days (b) 9 days
(c) 10 days (d) 11 days

14. A train running at a speed of 54 km/hour passes a signal post in 8 seconds. The length of train is
(a) 432 m.
(b) 150 m.
(c) 120 m.
(d) Data inadequate

15. Arshad can cover a circular path of radius 21 m. in 44 sec. He will cover a distance of 3 km in
(a) 16 mt. 20 secs.
(b) 16 mt. 40 secs.
(c) 18 mt. 00 secs.
(d) 18 mt. 30 sec

16. Two cars start from one point and move along two roads at right angles to each other. Their speeds are 36 km/hr and 48 km/hr respectively. After 15 sec. the distance between them will be
(a) 400 m. (b) 150 m.
(c) 300 m. (d) 250 m.

17. Rahim and Naeem and Hamid can do a work in 15 days, 6 days and 10 days respectively. All the three can together finish three times of that work in
(a) 21 days (b) 18 days
(c) 9 days (d) 15 days

18. A tank has a capacity of 240 litres. A pipe can empty 1/4th of the tank in 5 minutes and another pipe can empty 1/3rd of the tank in 6 minutes. The tank is filled and both the pipes are opened for 3 minutes. How much of the water is now left in the tank?
(a) 160 litres (b) 164 litres
(c) 196 litres (d) 200 litres

19. Amjad can row 18 km in 4 hrs down stream but while returning he takes 12 hours. The speed of the water current is
(a) 1.5 km/hr (b) 4 mm/hr
(c) 2 km/hr (d) 3 km/hr

20. B is twice as fast as A. If A can complete a job in 36 days, how long will it take for both A and B together to complete the same job?
(a) 24 days (b) 18 days
(c) 12 days (d) 9 days

21. Three pipes A, B and C can fill a cistern in 6 hours. After working together for 2 hours C is closed and A and B fill it in 7 hours. C alone can fill the cistern in
(a) 14 hrs (b) 21 hrs
(c) 10.5 hrs (d) 12 hrs

22. A thief steals a scooter at 1 P.M. and drives at the speed of 45 k/hr. The theft is discovered at 2 P.M. and the owner chases him at 54 km/hr. He will be caught at
(a) 7 P.M. (b) 6 P.M.
(c) 8 P.M. (d) 6.30 P.M.

23. A man rows upstream for 13 km and downstream for a distance of 28 km in 5 hrs each time. The speed of the water current is
(a) 2 km/hr (b) 1.5 km/hr
(c) 25 km/hr (d) 3 km/hr

24. A train with stoppages, covers a distance at 60 km/hr and without stoppages at 90 km/hr. The train stops for ... minutes per hour.
(a) 20 (b) 15
(c) 30 (d) 25

PERCENTAGE

IMPORTANT FORMULAE AND DEFINITIONS

1. Percentage means for every hundred, A fraction whose denominator is 100 is called a percentage while the numerator of the fraction is called the rate percent. Percentage is denoted by the sign (%), i.e., % sign stands for $\frac{1}{100}$. Addition of % sign with a number means division of the number by 100 or displacing the decimal sign to the left by two digits.

For example

300% = 3, 15% = 0.15 etc.

Removal of % sign means division by 100 or displacing the decimal sign to the left by two digits.

2. To convert a fraction into a percent, multiply the numerator and denominator by a number such that the denominator becomes 100. Then substitute the % sign in the numerator for 100 in the denominator, e.g.,

$\frac{1}{8}=12.5\%, \frac{3}{8}=37.5\%$ etc.

3. Two successive discounts of p% and q% allowed on a item are equivalent to a single discount of

$$\left(p+q-\frac{pq}{100}\right)\%$$

which is less that the sum of individual discounts, e.g., two discounts of 15% and 4% are equivalent to a single discount of

$$\left(15+4-\frac{15\times 4}{100}\right)=18.4\%$$

PROFIT AND LOSS

IMPORTANT FORMULAE AND DEFINITIONS

When a person deals in the purchase and sale of any item, he either gains or loses some amount generally. The commonly used terms in dealing with questions involving sale and purchase are:

Profit = S.P. – C.P. (If S.P. is more than C.P)

where S.P. stands for the selling price and C.P. for the cost price.

Loss = C.P. – S.P. (If C.P. is more than S.P.)

Profit and loss percent are always calculated with respect to the **cost price** of the item.

$$\text{Profit or loss\%} = \frac{\text{Profit or loss}}{\text{C.P.}} \times 100$$

Other important formulae are:

$$\text{C.P.} = \frac{100}{100+\%\text{ Profit}} \times \text{S.P.}$$

$$\text{C.P.} = \frac{100}{100-\%\text{Loss}} \times \text{S.P.}$$

$$\text{S.P.} = \frac{100+\%\text{ Profit}}{100} \times \text{C.P.}$$

$$\text{S.P.} = \frac{100-\%\text{ Loss}}{100} \times \text{C.P.}$$

PRACTICE PROBLEMS

1. **A bank exchanges British currency for Singapore currency at the rate of S$ 3.20 to £ 1. Calculate, in £, the amount exchanged for S$ 1,600 by a customer who also had to pay an extra 3% commission for this transaction.**
   (a) £ 475 (b) £ 485
   (c) £ 495 (d) £ 505

2. **A man bought a flat for Rs. 8,20,000. He borrowed 55% of this money from a bank. How much money did he borrow from the bank?**
   (a) Rs. 4,51,000 (b) Rs. 4,52,000
   (c) Rs. 4,53,000 (d) Rs. 4,54,000

3. **Mr. Rehman ordered a car worth Rs. 600,000 and was given a discount. Given that he paid Rs. 57,0000 for his new car, calculate the percentage discount he receive (d).**
   (a) 5 % (b) 7 %
   (c) 9 % (d) 10 %

4. **After spending 88% of his income, a man had Rs. 2160.00 left. Find his income.**
   (a) Rs. 18000 (b) Rs. 19000
   (c) Rs. 20000 (d) Rs. 22000

5. **The rent of a flat is Rs. 250 per month. The house tax is to be charged 10 percent yearly, but 12 percent of the rent is deducted for repairs and other expenses. The**

house tax is:
(a) Rs. 316 (b) Rs. 360
(c) Rs. 364 (d) Rs. 264

6. The population in a city increases at the rate of 5% annually. Its present population is 1,85,220. The population 3 years ago was
(a) 1,81,500 (b) 1,60,000
(c) 1,83,433 (d) 1,27,783

7. Hassan bought his home for Rs. 3,00,000 and sold it for Rs. 6,00,000. What was the percent of increase?
(a) 50 % (b) 200 %
(c) 100 % (d) 150 %

8. If $37\frac{1}{2}\%$ of a number is 45, then $87\frac{1}{2}\%$ of the number will be
(a) 105 (b) 102
(c) 96 (d) 92

9. The price of sugar is decreased by 10%. To restore to its original price, the new price must be increased by
(a) $9\frac{1}{11}\%$ (b) 10%
(c) 11% (d) $11\frac{1}{9}\%$

10. Nadeem spends 30% of his monthly income on food articles, 40% of the remaining on conveyance and clothes and saves 50% of the remaining. If his monthly salary is Rs. 18400, how much money does he save every month?
(a) Rs. 3864 (b) Rs. 4264
(c) Rs. 3624 (d) Rs. 5888

11. In an examination 450 candidates were boys and 550 were girls. If 32% of the boys and 38% of the girls passed the examination, the percentage of failed candidates is
(a) 35.3% (b) 62%
(c) 64.7% (d) 68%

12. In an examination, a candidate must secure 45% marks to pass. Azeem secures 325 marks but he fails by 35 marks. What were the maximum marks for the examination?
(a) 1800 (b) 900
(c) 800 (d) 750

13. From a container having pure milk, 20% is replaced by water and the process is repeated thrice. At the end of the third operation the milk is . . . . . pure.
(a) 45% (b) 56%
(c) 51.2% (d) 48.8%

14. When 40% of a number is added to 42, the result is the number itself. The number is
(a) 70 (b) 80
(c) 90 (d) 75

15. if $\frac{1}{8}$ is decreased by 25% we get
(a) 1/6 (b) 1/10
(c) 0.964 (d) 0.09375

16. A bookseller sold a book for Rs. 40.00 to make a profit of 15% percent. In order to earn a profit of 20%, he should have sold it for
(a) 41.74 (b) 43.75
(c) 42.25 (d) 40.60

17. How much would I have to pay for a book which cost Rs. 72 to produce, if the printing company sold it to a bookseller at 20% profit and in return the bookseller sold it to me at a profit of 25%?
(a) Rs. 104.00 (b) Rs. 106.00
(c) Rs. 108.00 (d) Rs. 110.00

18. A man buys a computer for Rs. 7,000 and sells it for Rs. 11,500. The percentage profit is nearly ..... percent:
(a) 54 (b) 58
(c) 64.3 (d) 70.6

19. **A dealer purchases 22 pencils for Rs. 20 and sells them at the rate of 10 pencils for Rs. 11. His profit percent is**
(a) 10% (b) 20%
(c) 21% (d) 22%

20. **The selling price of an article is Rs. 118 and the profit earned is 50%. The cost price of the article is**
(a) Rs. 68.70 (b) Rs. 77.00
(c) Rs. 78.66 (d) Rs. 80.00

21. **By selling a ceiling fan for Rs. 475, a dealer loses 5%. To get a gain of 5%, he should sell the fan for**
(a) Rs. 575.00 (b) Rs. 525.00
(c) Rs. 522.50 (d) Rs. 498.75

22. **Arshad sold a watch to Rahim at a gain of 5% and Arshad sold it to Rashid at a gain of 4%. If Rashid paid Rs. 1092 for it, the price paid by Arshad is**
(a) Rs. 993.72 (b) Rs. 996
(c) Rs. 1000 (d) Rs. 995.90

23. **The cost of 10 copies of a book equals the selling price of 8 copies. What is the percentage of gain or loss incurred?**
(a) 30% gain (b) 25% loss
(c) 25% gain (d) 30% loss

24. **Faisal bought 25 kg of wheat at the rate of Rs. 4.00 per kg and 35 kg at the rate of Rs. 4.50 per kg. He mixed the two and sold the mixture at the rate of Rs. 4.25 per kg. his gain/loss in this transaction is**
(a) Rs. 2.50 loss (b) Rs. 2.50 gain
(c) Rs. 5.00 (d) Rs. 5.00 gain
(e) None of these

25. **A fruit seller buys oranges at the rate of 12 for Rs. 20. For how much should he sell one orange to gain 20%?**
(a) Rs. 1.50 (b) Rs. 2
(c) Rs. 2.50 (d) Rs. 2.75

RATIO AND PROPORTION

IMPORTANT FORMULAE AND DEFINITIONS

Ratio: The relation which one quantity bears to another quantity is called the ratio between the two quantities. The quantities may be of same kind or different kinds, e.g., when we consider the ratio between 17 kg and 51 kg, we are considering the quantities of same kind and when we talk of allotting 2 lockers to 5 students of science, we are considering quantities of different kinds. Normally we consider the ratio between quantities of the same kind.

When the quantities are of the same kind, the ratio between them shown the number of times one quantity is contained in the other. The ratio between a and b where a and b are the measurement of the quantities is written as $\frac{a}{b}$ or a : b.

A ratio does not change if both of its terms are multiplied or divided by the same number. Thus

$$\frac{3}{5} = \frac{6}{10} = \frac{9}{15} \text{ etc.}$$

The values of a and b of the ratio are called the terms of the ratio, a is called the antecedent and b is called the consequent. The product of two or more ratios is called their compound ratio.

We can also write the relationship

between more than two quantities, e.g., the ratios between a : b and b : c can be written as a : b : c and the ratios a : b, b : c, c : d, can be written as a : b : c : d.

Proportion. If $\frac{a}{b} : \frac{c}{d}$, then a, b, c and d are said to be in proportion and we write a : b :: c : d.

This is read as a is to b as c is to d.

a : b :: c : d also means bc = ad.

From this relation we see that if any three of the four quantities are given, the fourth can be determined. Here a, b, c and d are called the first, second, third and fourth proportions.

If a : b :: b : c, then b is called the mean proportion between a and b and also $b^2 = ac$. In this case c is called the third proportion to a and b.

Proportional Parts. Proportion may be applied to divide a given number or quantity into a number of parts in a given ratio. These parts are called proportional parts and the process is known as proportional division.

(1) **Simple Partnership.** When the capitals of partners are invested for the same time, the partnership is called simple partnership. In such case profit or loss is distributed in proportion to the capital invested, expressed in the same denomination.

(2) **Compound Partnership.** When the capital, whether equal or unequal, is invested for different lengths of time, the partnership is called the **compound partnership**. In such cases profit or loss is distributed in proportion of the numbers representing the product of the capital and period of investment each expressed in same units of measurement.

PRACTICE PROBLEMS

1. **A certain solution is to be prepared by combining chemicals *X*, *Y* and *Z* in the ratio 18 : 3 : 2. How many litres of the solution can be prepared by using 36 litres of *X*?**
   (a) 46 litres (b) 47 litres
   (c) 45 litres (d) 49 litres

2. **A sum of money is divided among three persons. *X*, *Y* and *Z*, in the ratio 10 : 7 : 5. If *Y* gets Rs. 140 more than *Z*, how much will *X* get?**
   (a) Rs. 700.00 (b) Rs. 750.00
   (c) Rs. 735.00 (d) Rs. 875.00

3. **Six men earn as much as 7 women, 2 women as much as 3 boys and 4 boys as much as 5 girls. If a girl earns Rs. 400.00 a week what does a man earn in a day?**
   (a) Rs. 150.00 (b) Rs. 125.00
   (c) Rs. 200.00 (d) Rs. 175.00

4. **Three business partners, Q, R, and S, agree to divide their total profit for a certain year in the ratios 2:5:8 respectively. if Q's share was Rs. 4,000, what was the total profit of the business partners for the year?**
   (a) Rs. 26,000.00 (b) Rs. 30,000.00
   (c) Rs. 52,000.00 (d) Rs. 60,000.00

5. **A certain sum of money is distributed among two friends in the ration of 5 : 11. If one of them got Rs. 1200 more than the other, the total sum was**
   (a) Rs. 2000 (b) Rs. 2200
   (c) Rs. 3200 (d) Rs. 3600

6. **The ratio between the present ages of A and B is 4 : 5. If the difference between their ages is 8 years, what is the sum of their present ages?**
   (a) 32 years (b) 40 years
   (c) 65 years (d) 72 years

7. **In a class, the number of boys is more than the number of girls by 12% of the**

total strength of the class. The ratio of boys to girls is

(a) 17 : 14 (b) 14 : 11

(c) 25 : 28 (d) 28 : 25

8. **The scale of a map is 0.8 cm = 8.8 k. If the distance between two points on the map is 80.5 cm, the approximate distance between two points is**

(a) 9 km (b) 70 km

(c) 90 km (d) 880 km

9. **Rs. 3600 is divided among 60 men and women such that each man gets Rs. 75 more than each woman. The men got Rs. 1440 more than the women. The total number of women in the group is**

(a) 24 (b) 62

(c) 48 (d) 36

AVERAGE

AVERAGE OF TWO RATES.

1. In some problems, two or more rates must be averaged. When the times are the same for two or more different rates, add the rates and divide by the number of rates.

Example: If a man travels for 2 hours at 30 miles per hour, at 40 miles per hour for the next 2 hours, and at 50 miles per hour for the next 2 hours, then his average rate for the 6 hours is (30 + 40 + 50) ÷ 3 = 40 miles per hour.

2. When the times are not the same, but the distances are the same:

(a) Assume the distance to be a convenient length.

(b) Find the time at the first rate.

(c) Find the time at the second rate.

(d) Find the time at the third rate, if any.

(e) Add up all the distances and divide by the total time to find the average rate.

Example: A boy travels a certain distance at the rate of 20 miles per hour and returns at the rate of 30 miles per hour. What is his average rate for both trips?

Solution: The distance is the same for both trips. Assume that it is 60 miles. The time for the first trip is 60 ÷ 20 = 3 hours. The time for the second trip is 60 ÷ 30 = 2 hours. The total distance is 120 miles. The total time is 5 hours. Average rate is 120 ÷ 5 = 24 miles per hour.

Answer: The average rate is 24 miles per hour.

3. When the times are not the same and the distances are not the same:

(a) Find the time for the first distance.

(b) Find the time for the second distance.

(c) Find the time for the third distance, if any.

(d) Add up all the distances and divide by the total time to find the average rate.

Example: A man travels 100 miles at 20 miles per hour, 60 miles at 30 miles per hour, and 80 miles at 10 miles per hour. What is his average rate for the three trips?

Solution: The time for the first trip is 100 ÷ 20 = 5 hours. The time for the second trip is 60 ÷ 30 = 2 hours.

The time for the third trip is 80 ÷ 10 = 8 hours. The total distance is 240 miles. The total time is 1 hours. Average rate is 240 ÷ 15 = 16.

Answer: The average rate for the three trips is 16 miles an hour.

PRACTICE PROBLEM

1. **Rashid buys three books for Rs. 16.00 each and four books for Rs. 23.00 each, what will be the average price of books**
   (a) 18.00 (b) 20.00
   (c) 22.00 (d) 24.00

2. **A car travelled 100 km with half the distance at 40 km/h and the other half at 80 km/h. Find the average speed of the car for the whole journey.**
   (a) 53 Km/hr (b) 53.33 Km/hr
   (c) $54\frac{1}{4}$ Km/hr (d) 55 Km/hr

3. **The average age of a group of 13 boys is 13. When two more boys joined the group, the average rose by 2 years. The sum of the ages of the two new boys is**
   (a) 26 (b) 30
   (c) 50 (d) 56

4. **The average of 8 numbers is 12. If each number is increased by 2, the average of the new set of numbers is**
   (a) 12 (b) 13
   (c) 14 (d) 15

5. **A man drives for 5 hours at an average rate of 40 m.p.h. He develops some motor trouble and returns to his original starting point in 10 hours. What was his average rate on the return trip?**
   (a) 40 m.p.h. (b) 20 m.p.h.
   (c) 15 m.p.h. (d) 10 m.p.h.

6. **How many miles does a car travel if it averages at the rate of 35 miles per hour for 3 hours and 24 minutes?**
   (a) 109 (b) 110
   (c) 112 (d) 119

7. **A batsman in his 17th inning makes a score of 85, and thereby increases his average by 3. His average after the 17th inning is:**
   (a) 36 (b) 34
   (c) 37 (d) 35

8. **The average marks obtained by 22 candidates in an examination is 45. The average of the first ten is 55, while that of the last eleven is 40. The marks obtained by the 11 candidates are :**
   (a) 0 (b) 4
   (c) 3 (d) 9

9. **A train travels a distance of 40 km per hour on an average with stoppages and without stoppages its speed is 50 km per hour. How much time on an average per hour it stops.**
   (a) 15 min (b) 25 min
   (c) 12 min (d) 20 min

10. **A motorist travels for 3 hours at 40 miles per hour and then covers a distance of 80 miles in 2 hours and 40 minutes. His average rate for the entire trip was:**
    (a) 35.3 mph (b) 36 mph
    (c) 37 mph (d) 35.5 mph

EQUATIONS

1. If $a = 3b - c$, what is the value of b in terms of a and c?

(a) $a + c$ (b) $\frac{a+c}{3}$

(c) $\frac{a+c}{2}$ (d) $2(a + c)$

2. If $x - 4 = 11$, what is the value of $x - 8$?

(a) 15 (b) 8

(c) 7 (d) –7

3. If $2x - 5 = 98$, what is the value of $2x + 5$?

(a) 98 (b) 88

(c) 108 (d) 103

4. If $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$, what is a in terms of b and c?

(a) $\frac{b+c}{bc}$ (b) $\frac{bc}{b+c}$

(c) $b + c$ (d) $\frac{2bc}{b+c}$

5. If x is a positive number and $x^2 + 64 = 100$, what is the value of x?

(a) 6 (b) –6

(c) 12 (d) 36

6. If $2^{x+3} = 32$, what is the value of 3^{x+2}?

(a) 9 (b) 27

(c) 81 (d) 125

7. If $3a + 5b = 10$ and $5a + 3b = 30$, what is the average of a and b?

(a) 2.5 (b) 4

(c) 5 (d) 20

8. If $7x + 10 = 44$, what is the value of $7x - 10$?

(a) 24 (b) 34

(c) $4\frac{6}{7}$ (d) $-5\frac{5}{7}$

9. If $x - 4 = 9$, what is the value of $x^2 - 4$?

(a) 77 (b) 81

(c) 165 (d) 169

10. If $\frac{1}{3}x + \frac{1}{6}x + \frac{1}{9}x = 33$, what is the value of x

(a) 18 (b) 27

(c) 54 (d) 72

11. If $\frac{1}{a-b} = 5$ then a = _____.

(a) $b - 5$ (b) $b + \frac{1}{5}$

(c) $b - \frac{1}{5}$ (d) $b + 5$

12. Find the value of x if $3x - 2 = 2x + 3$

(a) 6 (b) 8

(c) 5 (d) 7

13. Find the value of x of

$\frac{1}{3}x - 2 = \frac{1}{16}$

(a) $8\frac{2}{5}$ (b) $7\frac{3}{16}$

(c) $6\frac{3}{16}$ (d) $6\frac{2}{7}$

14. Find the value of x if $3x - 4 = 7x - 24$

(a) 3 (b) 5

(c) 4 (d) 6

15. Find the value of x if $\frac{x}{2} = 3 + \frac{x}{4}$

(a) 11 (b) 10

(c) 12 (d) 8

16. Find the value of x if $\frac{x+2}{3} = \frac{x+1}{5}$

(a) $-\frac{7}{2}$ (b) $-\frac{6}{5}$

(c) $\frac{7}{2}$ (d) $\frac{6}{5}$

17. **Rabia bought 10 books, she gave 100 rupees to the shopkeeper. He returned her 2 rupees. For how much amount she bought one book?**
(a) 7.5 (b) 10
(c) 8.5 (d) 10.5

18. **If 3 is subtracted from 3 times of a number the remainder is 12. Find the number.**
(a) 2 (b) 3
(c) 5 (d) 4

19. **Fatima and Ayesha had 80 rupees in total. If Ayesha had four times that of Fatima's amount. Find their amounts.**
(a) Fatima 16 Ayesha 64
(b) Fatima 18 Ayesha 82
(c) Fatima 19 Ayesha 90
(d) Fatima 14 Ayesha 56

20. **If a certain number is added to five times of itself the sum is 36. Fin the number**
(a) 6 (b) 4
(c) 8 (d) 7

21. **Hira's amount is twice that if Madiha's amount. If they had 50 rupees more. The total amount becomes Rs. 125. How much amount each one is already having?**
(a) 22 (b) 30
(c) 25 (d) 27

INEQUALITIES

1. If $a < b$ and $b < c$ then
(a) $a > c$ (b) $a < c$
(c) $a \leq c$ (d) $a \geq c$

2. If $x > 0$, $y > 0$ and $x - y < 0$.
(a) $x > y$ (b) $y > x$
(c) $x \geq y$ (d) $y \geq x$

3. If $x + y < 9$ and $x - y < 7$, then the only possible value of x is _____
(a) $x < 9$ (b) $x > 9$
(c) $x < 8$ (d) $x > 8$

4. $b - a > 5$ and $b = 4x$ then
(a) $a < 1$ (b) $a > -1$
(c) $a > 1$ (d) $a < -1$

5. If $a > 3b-2$ and $a > b$, then which of the following statement is true
(a) $a = 1$ (b) $a < 1$
(c) $a > 1$ (d) $a \leq 1$

6. If $\frac{x}{3} - 4 > \frac{x}{2}$, then
(a) $x > -24$ (b) $-x < 24$
(c) $x = -24$ (d) $-x \leq 24$

7. What is the solution of the inequality $9x - 2 < 3x + 22$?
(a) $x = 4$ (b) $x > 4$
(c) $x < 4$ (d) $x \geq 4$

8. If $0 \leq a \leq 10$ and a be a whole number, then what is the solution for $a < 3$ and $a \geq 7$?
(a) 6 (b) 4
(c) 5 (d) no solution

9. If $4x > 3$ and $\frac{16}{2} x \leq 3$, then list all the integral values of x?
(a) {1,2,3,4}

(b) {5,6,7,8}
(c) {1,2,3,4,5,6}
(d) {1,2,3,4,5,6,7,8}

10. Find the solution set of $3x + 4 \leq 13$ $(x \in N)$
(a) {0, 1, 2, 3} (b) {1, 2, 3}
(c) {1, 2} (d) {0, 1, 2}

11. If $x \in W$, the solution set of $x < 4$ is
(a) {0, 1, 2, 3} (b) {0, 1, 2, 3, 4}
(c) {1, 2, 3, 4} (d) {1, 2, 3}

12. If $x \in W$, the solution set of $4x - 3 < 9$ is
(a) {0, 1, 2} (b) {1, 2}
(c) {0, 1, 2, 3} (d) {1, 2, 3}

13. Find the solution set of $3x + 4 \leq 13$ $(x \in N)$
(a) {0, 1, 2, 3} (b) {1, 2, 3}
(c) {1, 2} (d) {0, 1, 2}

14. Find the solution set of $2x + 1 < 5$ $(x \in W)$
(a) {0} (b) {1}
(c) {0, 1} (d) {0, 1, 2}

ANSWERS

REAL NUMBERS

1.	(c)	2.	(d)	3.	(c)	4.	(a)
5.	(a)	6.	(c)	7.	(d)	8.	(b)
9.	(b)	10.	(b)	11.	(a)	12.	(c)
13.	(d)	14.	(a)	15.	(a)	16.	(a)
17.	(c)	18.	(a)	19.	(b)	20.	(d)
21.	(a)	22.	(d)	23.	(b)	24.	(b)
25.	(b)	26.	(b)	27.	(a)	28.	(c)
29.	(d)	30.	(b)	31.	(c)	32.	(b)
33.	(a)	34.	(a)	35.	(b)	36.	(b)
37.	(b)	38.	(c)	39.	(c)	40.	(b)

SOLUTIONS

1. Such numbers are 103, 113, 123, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 143, 153, 163, 173, 183 and 193.

2. (d)

3. (c)

4. (a)

5. $(1011)^2 = (1000 + 11)^2$
$= 1000000 + 121 + 2 \times 11 \times 1000$
$= 1000000 + 121 + 22000$
$= 1022121$

6. $\frac{0.000434}{0.14} = \frac{434 \times 10^{-6}}{7 \times 2 \times 10^{-2}} = \frac{62}{2} \times 10^{-4}$
$= 31 \times 10^{-4} = 0.0031$

7. If x is the quotient then the number is $125x + 82$. When this number is divided by 25, we get
$\frac{125}{25}x + \frac{82}{25} = 5x + 3\frac{7}{25}$
Hence, the remainder will be 7.

8. $\frac{51.8}{18.5} = \frac{518}{185} = \frac{5180}{185} \times \frac{1}{10} = 2.8$

9. Let the number of days for which he works be x. Then number of days for which he was absent
$= 30 - x$
Wages for x days = Rs. 50x
Deduction for absent days
= Rs. $(30-x)10$
$1200 = 50x - (30 - x)10$
$= 50x - 300 + 10x$

Hence $60x = 1200 + 300 = 1500$

$\Rightarrow \quad x = 25$

Hence, he was absent for 5 days.

10. Since all the seven numbers are divisible by 4, in the sum of the numbers, 4 can be taken as a common factor, so the sum will definitely be divisible by 4.

11. Let the number of wrong question by x.

Then number of correct questions = $25 - x$ Total marks obtained

$= 2(25 - x) - \frac{1}{2}x = 37\frac{1}{2}$

$\Rightarrow 50 - 2x - \frac{1}{2}x = \frac{1}{2}x = \frac{75}{2}$

Thus, $\frac{5x}{2} = \frac{25}{2}$ or $x = 5$

TIME, DISTANCE AND WORK

1.	(b)	2.	(c)	3.	(c)	4.	(c)
5.	(d)	6.	(b)	7.	(a)	8.	(b)
9.	(d)	10.	(c)	11.	(a)	12.	(a)
13.	(c)	14.	(c)	15.	(b)	16.	(d)
17.	(c)	18.	(b)	19.	(a)	20.	(c)
21.	(a)	22.	(a)	23.	(b)	24.	(a)

SOLUTIONS:

1. $M_1 = 10, \quad T_1 = 14$ days

$M_2 = 4, \quad T_2 = ?$

Less Men, More Time

$M_2/M_1 = T_1/T_2$

M for Men and T for time

$\Rightarrow T_2 = \frac{M_1T_1}{M_2} = \frac{10 \times 14}{4} = 35$ days

---

2. $M_1 = 15, \quad T_1 = 10$ days

$M_2 = 8, \quad T_2 = ?$

= Less Men, more time

$= \frac{M_2}{M_1} = \frac{T_1}{T_2} \Rightarrow T_2 = \frac{M_1T_1}{M_2}$

$= \frac{15 \times 10}{8} = 18\frac{3}{4}$ days

---

3. Volume of Rectangle = L × W × H.

= Volume = $6 \times 5 \times 4 = 120m^3$

---

4. A: $t_1 = 10$days, B: $t_2 = 15$ days

A + B: t=?

$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{1}{6} \Rightarrow t = 6$ days

---

5. Zaheer = $t_1 = 80$ hrs.

Zaheer and son, $t_2 = 20$ hrs.

son, t=?

$\frac{1}{t} = \frac{1}{t_2} - \frac{1}{t_1} = \frac{1}{20} - \frac{1}{80} = \frac{4-1}{80} = \frac{3}{80}$

$t = 26\frac{2}{3}$ hrs.

---

6. $4m = 7B : 12M + 8B = 3(4M) + 8B$

$= 3(7B) + 8B$

$= 29B$

Boys	Days	
7	29	More boys, less days

$29x \qquad \frac{7}{29} = \frac{x}{29} \Rightarrow x = 7$

7.

Men	days
15	20
25	x

More men, less dys.

$\frac{15}{25} = \frac{x}{20} \Rightarrow x = 12$

---

8. $\frac{\text{Men}}{1200}$ $\frac{\text{days}}{24}$

$1200-300 = 900$ x

Less Men, More days

$\frac{1200}{900} = \frac{x}{24} \Rightarrow x = 32$

---

9. Normally portion of the cistern filled in one

hr = $\frac{1}{8}$

Due to leakage the portion filled per hour

= $\frac{1}{10}$

$\therefore$ Leakage per hour $\frac{1}{8}-\frac{1}{10}=\frac{5-4}{40}=\frac{1}{40}$

Hence the leak shall empty it in 40 hours.

10. (c) Let the distance between the two stations be d km.

Then, the time taken to travel d km at

40 km/hr = $\frac{d}{40}$ hrs.

Time taken at 30 km/hr = $\frac{d}{30}$ hrs.

Difference in time = $\frac{d}{30}-\frac{d}{40}$ hrs = $\frac{6}{60}$ hrs

Thus $\frac{1}{10} = \frac{4d-3d}{120}=\frac{d}{120}$ or $d=12$km

11. As the hole leaks out 10% of water, it means pipe will fill 90% of the tank in 15 hours, So it will fill the complete tank in

$\frac{15\times100}{90}=16\frac{2}{3}$ hrs $= 16$ hrs 40 min.

12. Let the distance he has to travel be d km. Time taken to travel this distance at 4 km/hr

$= \frac{d}{4}$ hrs

Time taken at 5 km/hr = $\frac{d}{5}$ hrs

Difference in time $= \frac{d}{4}-\frac{d}{5}=\frac{d}{20}$

Actual difference = 15 min. = $\frac{1}{4}$ hrs

$\frac{d}{20}=\frac{1}{4}=$ or $d=5$ km

13. (c)

14. Speed of the train = 54 km/hr

$= \frac{54\times1000}{3600} = 15$ m/s

$\therefore$ Length of the train = 15 x 8 = 120 metres

15. Distance covered = 2 x π x 21 m

$= 2 \times \frac{22}{7} \times 21 = 132$ m

$\therefore$ His speed $= \frac{132}{44} = 3$ m/sec.

Time taken to cover 3 km

$= \frac{3000}{3}$ sec = 16 min. 40 sec.

16. In 15 sec. the first car will travel a distance of

$\frac{36\times1000}{60\times60}$ x 15 = 150 m.

The second car will travel

$\frac{48\times1000}{60\times60}$ x 15 = 200 m.

As they moved in directions at 90° to each other, the distance between them

$= \sqrt{200^2+150^2} = 50\sqrt{4^2+3^2}$

$= 50 \times 5 = 250$ m

17. **The work they can do together in a day is**

$$\frac{1}{15}+\frac{1}{6}+\frac{1}{10}=\frac{2+5+3}{30}$$

$$=\frac{10}{30}=\frac{1}{3}$$

Thus together they can finish the work in 3 days and they can finish 3 times that work in 3 x 3 = 9 days

18. As the pipes are opened for 3 minutes, in 3 min, A can empty $\frac{1}{4}x\frac{1}{5}x3=\frac{3}{20}$ of the tank

B can empty $\frac{1}{3}x\frac{1}{6}x3=\frac{1}{6}$ of the tank

Thus, the part of the tank emptied in 3 min

$$=\frac{3}{20}+\frac{1}{6}=\frac{9+10}{60}=\frac{19}{60}$$

Part of the tank filled = $\frac{41}{60}$

Hence, the amount of water left in the tank

$=\frac{41}{60}x240=164$ litres

19. If x is his speed in still water and y the speed of the current, then

$$x+y=\frac{18}{4}=\frac{9}{2}$$

Also, $x-y\ \frac{18}{12}=\frac{3}{2}$

Solving these equations, we get

$$2y=\frac{6}{2}=3 \text{ or } y=\frac{3}{2} \text{ km/hr}$$

$= 1.5$ km/hr

20. When a can finish the job in 36 days and B is twice as fast as A, it means he can finish the job in half the number of days, i.e., in 18 days.

When they work together, their one day's work

$$=\frac{1}{36}+\frac{1}{18}=\frac{3}{36}=\frac{1}{12}$$

Hence they can finish the job in 12 days.

21. Part of the cistern (A + B + C) can fill in 1 hr

$$=\frac{1}{6}$$

In 2 hours they will fill = $2x\frac{1}{6}=\frac{1}{3}$

Part of the cistern unfilled = $1-\frac{1}{3}=\frac{2}{3}$

A and B fill it in 7 hours.

$\therefore$ (A + B) can fill in one hour

$\frac{1}{7}x\frac{2}{3}=\frac{2}{21}$ of the cistern

Hence C can fill in one hour

$$=\frac{1}{6}-\frac{2}{21}=\frac{7-4}{42}=\frac{3}{42}=\frac{1}{14}$$

Thus, C will fill the cistern in 14 hrs.

22. Suppose the thief is caught in x hours after the theft is noticed.

Distance traveled by the thief in (x + 1) hr

$= 45 (x + 1)$ km

Distance traveled by the owner = 54x

$\therefore$ $54x = 45x + 45$ or $9x = 45$

$\Rightarrow$ $x = 5$

Thus, the thief will be caught at 7 P.M.

23. Let x be the speed of the boat and y that of the stream.

Then $x+y=\frac{28}{5}$

and $x-y=\frac{13}{5}$

Simplifying, we get

$$2y=\frac{15}{5}=3$$

or $y = 1.5$ km/hr

24. The train traveled 90 km without stoppage and 60 km with stoppage. Thus, it stops for the time equivalent to the time required to cover 30 km.

$= \frac{30}{90} = \frac{1}{3}$ hr = 20 min.

PERCENTAGE

1.	(b)	2.	(a)	3.	(a)	4.	(a)
5.	(d)	6.	(b)	7.	(c)	8.	(a)
9.	(d)	10.	(a)	11.	(c)	12.	(c)
13.	(c)	14.	(a)	15.	(d)	16.	(a)
17.	(c)	18.	(c)	19.	(c)	20.	(c)
21.	(b)	22.	(c)	23.	(c)	24.	(a)
25.	(a)						

SOLUTIONS:

1. $1S\$ = £\frac{1}{3.20}$

$= S\$1600 = \frac{1}{3.20} \times 1600 = £\,500$

Extra Tax @3% $= £500 \times \frac{3}{100} = £15$

Amount Exchanged = 500 - 15 = £ 485

---

2. Cost of Flat = Rs. 8,20,000

55% of the amount is borrowed by the bank, so Amount borrowed = $8,20,000 \times \frac{55}{100}$

= Rs. 4,51,000

---

3. Original Price = P_o = Rs. 6,00,000

Paid Price P_1 = Rs. 5,70,000

% Discount = ? Dicsount = P_1-P_2 = 30,000

$= \frac{30000}{600000} \times 100 = 5\%$

4. Rate of spending = r = 88%

Left Amount = Rs. 2160

Let Income = x

$x(1-r) = 2160 \Rightarrow x(1-\frac{88}{100}) = 2160$

$\frac{12}{100}x = 2160 \Rightarrow \frac{2160}{12} \times 100 = 7 = 18000$

5. Monthly Rent = Rs. 250

Yearly " $= 250 \times 12$ = Rs. 3000

Misc. @ 12% p.o. $= 3000 \times 12 = 360$

Remaining = 3000 - 360 = 2640

Tax @ 10% p.a. $= 2640 \times \frac{10}{100}$ = Rs. 264

6. $P = Po(1+\frac{r}{100})^{-t} = 1,85,220\,(1+\frac{5}{100})^{-3}$

= 1,60,000 For ago, t is -ve

7. C = Rs. 3,00,000, S = Rs. 6,00,000

Increase = S - C = Rs. 3,00,000

% increase on $C = \frac{300000}{300000} \times 100 = 100\%$

8. Let the number be x, then

$37\frac{1}{2}\%$ of $x = 45$

or $\frac{75}{2} x \frac{1}{100} x\ x = 45$

or $\frac{3}{8} x\ x = 45$

$\Rightarrow x = \frac{45 \times 8}{3} = 120$

Then,

$87\frac{1}{2}\%$ of $120 = \frac{175}{2} x \frac{1}{100} x 120 = 105$

9. Let the price of the sugar be Rs. 100

Decreased price = Rs. 90

To restore to the original price the increase should be Rs. 10

$\therefore$ % increase $= \frac{10}{90} x 100 = \frac{100}{9} = 11\frac{1}{9}\%$

10. His monthly income = Rs. 18400

Amount spent on food = 30% of 18400

= Rs. 5520

Balance left = 18400 – 5520 = Rs. 12880

Amount spent on conveyance and clothes

= 40% of 12880 = Rs. 5152

Balance left = 12880 – 5152 = Rs. 7728

Amount saved = 50% of 7728 = Rs. 3864

11. Number of boys = 450

Number of boys who passed

$$= \frac{32}{100} \times 450 = 144$$

Number of girls who passed the examination

$$= \frac{38}{100} \times 550 = 209$$

Number of boys and girls who failed

= (450 + 550) – (144 + 209)

= 1000 – 353 = 647

$$\% \text{ of failed candidates} = \frac{647}{1000} \times 100$$

= 64.7

12. Let the maximum marks be x

pass marks = 45% of x = 0.45x

$\therefore$ 325 = 0.45x – 35

or 0.45x = 360

$$\Rightarrow \quad x = \frac{360}{0.45} = 800$$

13. Let there be 100 litres of milk in the container

After the first operation, the composition is

Milk = 80 litres and water = 20 litres

In 2nd operation when 20% of the mixture is taken out, milk removed is 16 litres.

So after the second operation, the amount of milk = 64 litres

and water = 100 – 64 = 36 litres

Similarly after 3rd operation, the amount of milk in the container

= 64 – 12.8 = 51.2 litres

Hence purity of the milk is 51.2%

14. Let the number be x

$$40\% \text{ of } x = \frac{40}{100}x$$

$$x = \frac{40}{100}x + 42 = \frac{2}{5}x + 42$$

$\Rightarrow$ 5x = 2x + 210

or 3x = 210 or x = 70

15. $$25\% \text{ of } \frac{1}{8} = \frac{25}{100} \times \frac{1}{8} = \frac{1}{32}$$

Therefore the required number is

$$\frac{1}{8} + \frac{1}{32} = \frac{3}{32} = 0.09375$$

16. Here $S_1 = 40$, $P_1 = 1.150$

$S_2 = ?$, $P_2 = 1.20$

$$S_2 = \frac{P_2}{P_1} \times S1 = \frac{1.20}{1.15} \times 40 = 71.74$$

Note: $\frac{S_1}{S_2} = P_1/P_2$

17. List Price $(1 \pm r_1)(1 \pm r_2) \ldots$ = Net Amount Payable

$$72\left(1 + \frac{20}{100}\right)\left(1 + \frac{25}{100}\right) = \text{Rs. } 108$$

18. C = 7000, S = 11,500

$$P = \frac{S}{C} = \frac{11500}{7000} = 1.643$$

%P = 64.30%

19. Cost price of 22 pencils = Rs. 20

Selling price of 10 pencils = Rs. 11

Selling Price of 22 pencils = Rs. $\frac{11}{10} \times 22$

$$= \text{Rs. } \frac{121}{5}$$

Profit = S.P.–C.P. = $\frac{121}{5} - 20 = \text{Rs. } \frac{21}{5}$

Percentage profit= $\frac{21}{5} \times \frac{1}{20} \times 100 = 21\%$

20. Let C.P. = x; Profit = 50%

$$\text{S.P.} = x + \frac{50}{100}x = \frac{3x}{2}$$

But S.P. = Rs. 118

$$\therefore \quad 118 = \frac{3x}{2} \text{ or } x = \frac{236}{3} = \text{Rs. } 78.66$$

21. S.P. = Rs. 475, Loss = 5%

$$\therefore \text{ C.P.} = \frac{100}{100-5} \times 475 = \text{Rs. } 500$$

In second case, gain = 5%

$$\therefore \quad \text{S.P.} = \frac{100+5}{100} \times 500 = 525.$$

22. Let Arshad's C.P. = Rs. 100

Then Rahim's C.P. = Rs. 105

Rashid C.P.'s = Rs. (105 + 4.20) = 109.20

Hence price paid by Arshad

$$= \frac{100}{109.20} \times 1092 = \text{Rs. } 1000$$

23. Let the C.P. of one book be Rs. x

C.P. of 10 books = 10x

S.P. of 8 books = 10x

$$\text{S.P. of 10 books} = \frac{10x}{8} \cdot 10x = \frac{25}{2}x$$

$$\text{Profit} = \frac{25}{20}x - 10x = \frac{5}{2}x$$

$$\text{Profit \%} \quad \frac{5}{2}x \times \frac{1}{10x} \times 100 = 25\%$$

24. Cost price for 25 kg=25x4 = Rs. 100

Cost price for 35 kg = 35x4.5 = Rs. 157.50

Total cost price for 60 kg = Rs. 257.50

S.P. for 60 kg = 60 x 4.25 = Rs. 255

∴ Loss = 257.50 – 255.00 = Rs 2.50

25. C.P. for 12 oranges = Rs. 20, gain = 20%

$$\text{S.P. for 12 oranges} = \frac{100+20}{100} \times 20$$

= Rs. 24

Therefore S.P. for one oranges=Rs. 2

RATIO & PROPORTION

1. (a) 2. (a) 3. (b) 4. (b)
5. (c) 6. (d) 7. (b) 8. (d)
9. (d)

SOLUTIONS

1. X : Y : Z

18 : 3 : 2, Sum = 23

Amount of soln. if x = 36 litres

$$= \frac{23}{18} \times 36 = 46 \text{ litres}$$

2. Ratio: $\frac{x}{10} : \frac{y}{7} : \frac{z}{5}$

Amount: a : a : a

10a : 7a : 5a

= 7a - 5a = 140

2a = 140 = a = 70

= X's = 10 x 70 = 700

3. 6M = 7W; 2W = 3B: 4B = 5G

Let earning of a girl per day = Rs. x

(By successive reverse substitution)

$$M = \frac{7}{6} \times \frac{3}{2} \times \frac{5}{4} \times 400 = 875$$

$$\text{M's one day earning} = \frac{875}{7} = \text{Rs. } 125$$

4. $\frac{Q}{2} : \frac{R}{5} : \frac{S}{8}$ $\frac{\text{Sum}}{15}$

If share of Q is 2, total profit = 15

" " 1, " $= \frac{15}{2}$

EQUATIONS

1.	(b)	2.	(c)	3.	(c)	4.	(b)
5.	(a)	6.	(c)	7.	(a)	8.	(a)
9.	(c)	10.	(c)	11.	(b)	12.	(c)
13.	(c)	14.	(b)	15.	(c)	16.	(a)
17.	(a)	18.	(c)	19.	(a)	20.	(a)

21. ©

SOLUTIONS:

1. b. $a = 3b - c$

$\Rightarrow 3b = a + c$

$\Rightarrow b = \frac{a + c}{3}$

2. c. $x - 4 = 11$

$\Rightarrow x = 11 + 4 = 15$

$x - 8 = 15 - 8 = 7$

3. c. $2x - 5 = 98$

$\Rightarrow 2x - 5 + 10 = 98 + 10$

$\Rightarrow 2x + 5 = 108$

4. b. $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$

$\Rightarrow \frac{1}{a} = \frac{b + c}{bc}$

$\Rightarrow a = \frac{bc}{b+c}$

5. a. $x^2 + 64 = 100$

$\Rightarrow x^2 = 100 - 64 = 36$

$x = 6$

as x is positive

6. c. $2^{x+3} = 32$

$\Rightarrow 2^{x+3} = 2^5$

$\Rightarrow x + 3 = 5$

$\Rightarrow x = 5 - 3 = 2$

$3^{x+2} = 3^{2+2} = 3^4 = 81$

7. a. $3a + 5b = 10$

$5a + 3b = 30$

Adding $8a + 8b = 40$

Dividing by 8, $a + b = 5$

Average of a and b

$= \frac{a+b}{2} = \frac{5}{2} = 2.5$

8. a. $7x + 10 = 44$

$\Rightarrow 7x + 10 - 20 = 44 - 20$

$\Rightarrow 7x - 10 = 24$

9. c. $x - 4 = 9$

$\Rightarrow x = 9 + 4 = 13$

$x^2 - 4 = (13)^2 - 4$

$= 169 - 4 = 165$

10. c. $\frac{1}{3}x + \frac{1}{6}x + \frac{1}{9}x = 33$

Multiply by 18

$6x + 3x + 2x = -594$

$11x = 594$

$\Rightarrow x = 54$

11. b. $\frac{1}{a-b} = 5$

$\Rightarrow 5a - 5b = 1$

$\Rightarrow 5a = 1 + 5b$

$\Rightarrow a = \frac{1}{5} + b$

12. $3x - 2 = 2x + 3$

or $3x - 2 + 2 = 2x + 3 + 2$

or $3x = 2x + 5$

or $3x - 2x = 2x + 5 - 2x$

$x = 5$

$\therefore$ Solution set = { 5 }

13. $3 \times \frac{1}{3}x - 3 \times 2 = 3 \times \frac{1}{16}$

$x - 6 = \frac{3}{16}$

$x = \frac{3}{16} - \frac{6}{1}$

$x = \frac{3 + 96}{16}$

$x = \frac{99}{16}$

$x = 6\frac{3}{16}$

14. $3x - 4 = 7x - 24$

$3x - 4 + 4$	$= 7x - 24 + 4$
$3x$	$= 7x - 20$
$3x - 7x$	$= 7x - 20 - 7x$
$-4x$	$= -20$
$\frac{-4x}{-4}$	$= \frac{-20}{-4}$

$x = 5$

15. $4 \times \frac{x}{2} = 4 \times 3 + \frac{4x}{4}$

$2x = 12 + x$

$2x - x = 12 + x - x$

$\Rightarrow$ Rashid's age = 60 years

$\Rightarrow$ Zahid's age = 40 years

29. a. $f(2) = 4(2)^3 - 2(2) + 1$

$= 32 - 4 + 1 = 29$

30. c. $P(-2) = 2(-2)^3 + 2(-2)^2 - 1$

$= -16 + 8 - 1 = -9$

31. d. $\frac{x^2 - y^2}{x - y} = \frac{(x - y)(x + y)}{x - y} = x + y$

$= 117 = 118 = 235$

32. c. $(2x + 3)(x + 6) - (2x - 5)(x + 10)$

$= 2x^2 + 12x + 3x + 18$

$- (2x^2 + 20x - 5x - 50)$

$= 2x^2 + 15x + 18 - 2x^2 - 15x + 50$

$= 68$

33. b. $\left(\frac{1}{x} + x\right)^2 = 100$

$\Rightarrow \frac{1}{x^2} + x^2 + 2 = 100$

$\Rightarrow \frac{1}{x^2} + x^2 = 100 - 2 = 98$

34. a. $(x - y)^2 = 2$

$\Rightarrow x^2 + y^2 - 2xy = 2$

$\Rightarrow 4 - 2xy = 2$

$\Rightarrow -2xy = 2 - 4 = -2$

$\Rightarrow xy = 1$

35. d. $(x - 6)(x - 4)$

$= x^2 - 4x - 6x + 24$

$= x^2 - 10x + 24$

36. c. $(a + b)^2 + (a - b)^2$

$= a^2 + b^2 + 2ab + a^2 + b^2 - 2ab$

$= 2a^2 + 2b^2$

37. b. $(x + y)^2 = (8)^2$

$\Rightarrow x^2 + y^2 + 2xy = 64$

$\Rightarrow x^2 + y^2 + 30 = 64$

$\Rightarrow x^2 + y^2 = 34$

38. c. $\left(x - \frac{1}{x}\right)^2 = (3)^2$

$\Rightarrow x^2 + \frac{1}{x^2} - 2 = 9$

$\Rightarrow x^2 + \frac{1}{x^2} = 11$

39. d. $4ab = (a + b)^2 - (a - b)^2$

$= (15)^2 - (3)^2$

$= 225 - 9 = 216$

EQUATIONS

1.	(b)	2.	(c)	3.	(c)	4.	(b)
5.	(a)	6.	(c)	7.	(a)	8.	(a)
9.	(c)	10.	(c)	11.	(b)	12.	(c)
13.	(c)	14.	(b)	15.	(c)	16.	(a)
17.	(a)	18.	(c)	19.	(a)	20.	(a)

21. ©

SOLUTIONS:

1. b. $a = 3b - c$

$\Rightarrow 3b = a + c$

$\Rightarrow b = \frac{a + c}{3}$

2. c. $x - 4 = 11$

$\Rightarrow x = 11 + 4 = 15$

$x - 8 = 15 - 8 = 7$

3. c. $2x - 5 = 98$

$\Rightarrow 2x - 5 + 10 = 98 + 10$

$\Rightarrow 2x + 5 = 108$

4. b. $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$

$\Rightarrow \frac{1}{a} = \frac{b + c}{bc}$

$\Rightarrow a = \frac{bc}{b+c}$

5. a. $x^2 + 64 = 100$

$\Rightarrow x^2 = 100 - 64 = 36$

$x = 6$

as x is positive

6. c. $2^{x+3} = 32$

$\Rightarrow 2^{x+3} = 2^5$

$\Rightarrow x + 3 = 5$

$\Rightarrow x = 5 - 3 = 2$

$3^{x+2} = 3^{2+2} = 3^4 = 81$

7. a. $3a + 5b = 10$

$5a + 3b = 30$

Adding $8a + 8b = 40$

Dividing by 8, $a + b = 5$

Average of a and b

$= \frac{a+b}{2} = \frac{5}{2} = 2.5$

8. a. $7x + 10 = 44$

$\Rightarrow 7x + 10 - 20 = 44 - 20$

$\Rightarrow 7x - 10 = 24$

9. c. $x - 4 = 9$

$\Rightarrow x = 9 + 4 = 13$

$x^2 - 4 = (13)^2 - 4$

$= 169 - 4 = 165$

10. c. $\frac{1}{3}x + \frac{1}{6}x + \frac{1}{9}x = 33$

Multiply by 18

$6x + 3x + 2x = 594$

$11x = 594$

$\Rightarrow x = 54$

11. b. $\frac{1}{a-b} = 5$

$\Rightarrow 5a - 5b = 1$

$\Rightarrow 5a = 1 + 5b$

$\Rightarrow a = \frac{1}{5} + b$

12. $3x - 2 = 2x + 3$

or $3x - 2 + 2 = 2x + 3 + 2$

or $3x = 2x + 5$

or $3x - 2x = 2x + 5 - 2x$

$x = 5$

$\therefore$ Solution set = { 5 }

13. $3 \times \frac{1}{3}x - 3 \times 2 = 3 \times \frac{1}{16}$

$x - 6 = \frac{3}{16}$

$x = \frac{3}{16} - \frac{6}{1}$

$x = \frac{3 + 96}{16}$

$x = \frac{99}{16}$

$x = 6\frac{3}{16}$

14. $3x - 4 = 7x - 24$

$3x - 4 + 4 = 7x - 24 + 4$

$3x = 7x - 20$

$3x - 7x = 7x - 20 - 7x$

$-4x = -20$

$\frac{-4x}{-4} = \frac{-20}{-4}$

$x = 5$

15. $4 \times \frac{x}{2} = 4 \times 3 + \frac{4x}{4}$

$2x = 12 + x$

$2x - x = 12 + x - x$

x = 12

16. $\frac{15(x+2)}{3} = \frac{15(x+1)}{3}$

$5(x+2) = 3(x+1)$

$5x + 10 = 3x + 3$

$5x + 10 - 3x = 3x + 3 - 10$

$2x + 10 = 3$

$2x + 10 - 10 = 3 - 10$

$2x = -7$

$X = -\frac{7}{2}$

17. Let the price of one book by = x rupees

Price of 10 such books = 10 x rupees

Total amount she gave = 100 rupees

Balance = 25 rupees

According to the statement

$10x + 25 = 100$

$10x = 100 - 25$

$10x = 75$

$x = \frac{75}{10} = 7.5$ rupees.

18. Let the number is = x

Three times the number = 3x

3 subtracted from it = 3x – 3

According to the statement of the question;

$3x - 3 = 12$

$3x = 12 + 3$

$3x = 15$

$\frac{1}{3} \times 3x = \frac{15}{3}$

$x = 5$

19. Let Fatima's amount = x rupees

Ayesha's amount be = 4x

Total Amount = x + 4x

According to the given statement;

$x + 4x = 80$ rupees

$5x = 80$ rupees

$\frac{1}{5} \times 5x = \frac{1}{5} \times 80$ rupees

$= 16$ rupees

So Fatima had Rs. 16 and Ayeshad had

Rs. 4 x 16 = Rs. 64

(Check 16 + 64 = 80)

21. Let the number = x

Five times the number = 5x

According to statement

$X + 5x = 36$

$6x = 36$

$\frac{6x}{6} = \frac{36}{6}$

$x = 6$

21. Let Madiha's amount = Rs. X

Hira's amount = 2x

According to statement

$x + 2x + 50 = 125$

$3x + 50 = 125$

$3x + 50 - 50 = 125 - 50$

$3x = 75$

$\frac{3x}{3} = \frac{75}{3}$

$x = 25$

INEQUALITIES

1. (b) 2. (b) 3. (c) 4. (b)
5. (c) 6. (a) 7. (c) 8. (d)
9. (c) 10. (b) 11. (a) 12. (a)
13. (b) 14. (c)

SOLUTIONS:

3. $x + y < 9$

$x - y < 7$

$2x < 16$

$$\frac{2x}{2} < \frac{16}{2}$$

$x < 8$

4. Put b = 4, then $4 - 1 < 5$

$-a > 5 - 4$

$-a > 1$

$\Rightarrow \quad -a > 1$

5. $a > 3b - 2$

$a > 3a - 2 \ (\because a > b)$

$a - 3a > -2$

$-2a > -2$

$\frac{-2a}{-2} > \frac{-2}{-2} \rightarrow a > 1$

6. $\frac{x}{3} - 4 > \frac{x}{2}$

$\frac{x}{3} - 4 - \frac{x}{2} > \frac{x}{2} - \frac{x}{2}$

$\frac{x}{3} - \frac{x}{2} - 4 + 4 > 4$

$\frac{x}{3} - \frac{x}{2} > 4$

$\frac{-x}{6} > 4$

$-x > 24$

$X > -24$

7. $9x - 2 < 3x + 22$

$9x - 2 - 3x < 3x - 3x + 22$

$6x - 2 + 2 < 22 + 2$

$6x < 24$

$\frac{6x}{6} < \frac{24}{6}$

$x < 4$

8. Given set is $0 \le a \le 10$ i.e. {0,1,2,3,4,5,6, 7,8,9,10}

and $a < 3$ are {0,1,2} and $a \ge 7$ are {7,8,9,10}. Therefore, there are no common elements. So, there is no solution for the given inequalities.

9. $4x > 3 \ , \ \frac{1}{2} x \le 3$

$x > \frac{3}{4} \ , \ \frac{1}{2} \times x2 \le 3 \times 2$

$x \le 6$

Since, the value of x (integer) is greater than $\frac{3}{4}$ and less than and eaual to 8 are {1,2,3,4,5,6}

10. b. $3x + 4 \le 13$

$\Rightarrow 3x \le 13 - 4$

$\Rightarrow 3x \le 9$

$\Rightarrow x \le 3$

As $x \in N$, solution set is {1, 2, 3}

11. a. As $x \in W$ and $x < 4$

Solution set is {0, 1, 2, 3}

12. a. $4x - 3 < 9$

$\Rightarrow 4x < 9 + 3$

$\Rightarrow 4x < 12$

$\Rightarrow x < 3$

As $x \in W$ solution set is {0, 1, 2}

13. b. $3x + 4 \le 13$

$\Rightarrow 3x \le 13 - 4$

$\Rightarrow 3x \le 9$

$\Rightarrow x \le 3$

As $x \in N$, solution set is {1, 2, 3}

14. c. $2x + 1 < 5$

$\Rightarrow 2x < 5 - 1$

$\Rightarrow 2x < 4$

$\Rightarrow x < 2$

As $x \in W$ solution set is {0, 1}