1. Evaluate the limit of $\frac{1-\sec^2 x}{\cos x - 1}$ as x approaches 0.

Ans: -2

Solution:

$$\frac{1-\sec^2 x}{\cos x - 1} = \frac{-2\sec x(-\sec x \tan x)}{-\sin x}$$
$$= \frac{-2\sec^2 x \tan x}{\sin x}$$
$$= \frac{-2\sin x}{\cos x \cos^2 x \text{Sin}x}$$
$$= \frac{-2}{\cos^3 x} = -\frac{2}{\cos 0^\circ} = -2$$

Suggested Solution using calculator: (Mode Radian)

$$= \frac{1-\left[\frac{1}{\cos(0.0001)}\right]^2}{\cos(0.0001)-1}$$
$$= -2$$

2. Evaluate the limit of $\frac{4\tan^3 x}{2\sin x - x}$ as x approaches 0.

Ans: 0
Solution:

$$\frac{4\tan^3 x}{2\text{Sin } x - x} = \frac{4(3)\tan^2 x \sec^2 x}{2 \text{ Cos } x - 1}$$
$$= \frac{12 \text{ Sin}^2 x}{\text{Cos}^2 x \text{Cos}^2 x(2 \text{ Cos } x - 1)}$$
$$= \frac{0}{1(2-1)} = \frac{0}{1}$$
$$= 0$$

Suggested Solution using calculator: (Mode Radian)

$$= \frac{4(\tan 0.0001)^3}{2\sin 0.0001 - 1}$$
$$= 0$$

3. Evaluate $\lim_{x\to 0} \frac{x+\text{Sin}2x}{x-\text{Sin}2x}$

Ans: -3

Solution:

$$\lim_{x\to 0}\frac{x+\text{Sin}2x}{x-\text{Sin}2x} = \frac{1+2\text{Cos}2x}{1-2\text{ Cos}2x}$$
$$= \frac{x+\text{Sin}2x}{x-\text{Sin}2x} = \frac{1+2}{1-2} = -3$$

Suggested Solution using calculator: (Mode Radian)

$$= \frac{(0.0001)+\sin[2(0.0001)]}{(0.0001)-\sin[2(0.0001)]}$$
$$= 0$$

4. Evaluate: $\lim_{x\to 0}\frac{\text{In Sin } x}{\text{In Tan } x}$

Ans: 1
Solution:

$$\lim_{x\to 0}\frac{\text{In Sin } x}{\text{In tan } x} = \frac{\text{Cos}/\text{Sin } x}{\text{Sec}^2 x/\tan x}$$
$$\lim_{x\to 0}\frac{\text{In Sin } x}{\text{In tan } x} = \frac{\text{Cos } x \text{ Sin} x}{\text{Sin } x \text{ Cos } x \text{ Sec}^2 x}$$
$$\lim_{x\to 0}\frac{\text{In Sin } x}{\text{In tan } x} = \text{Cos}^2 x$$
$$\lim_{x\to 0}\frac{\text{In Sin } x}{\text{In tan } x} = \text{Cos}^2 0^\circ = 1$$

5. What is the allowable error in measuring the edge of a cube that is intended to hold 8 cu. m. if the error in the computed volume is not to exceed 0.03 m^3.
Ans: 0.0025

Solution:

$$V = x^3$$
$$dV = 3x^2 dx$$
$$8 = x^3$$
$$x = 2$$
$$dV = 3x^2 dx$$
$$0.03 = 3(2)^2 dx$$
$$dx = 0.0025$$

6. A surveying instrument is placed at a point 180 m. from the base of a bldg. on a level ground. The angle of elevation of the top of the bldg. is 30 degrees as measured by the instrument. What would be error in the height of the bldg. due to an error of 15 minutes in this measured angle by using differential equation?
Ans: 1.05 m.

Solution:

$$h = 180 \tan\theta$$
$$dh = 180 \sec^2\theta \, d\theta$$
$$\text{when } \theta = 30^\circ$$
$$\text{Sec } 30^\circ = 1.1547$$
$$\text{Sec}^2 30^\circ = 1.333$$
$$d\theta = \frac{15'}{60} 0.25^\circ$$
$$d\theta = \frac{0.25\pi}{180}$$
$$d\theta = 0.00436 \text{ radians}$$
$$dh = 180(1.333)(0.00436)$$
$$dh = 1.05\text{m.}$$

7. If the area of the circle is $64\pi \text{ mm}^2$, compute the allowable error in the area of a circle if the allowable error in the radius is 0.02 mm.
Ans: 1.01 mm^2

Solution:

$A = \pi r^2$

$64\pi = \pi r^2$

$r = 8mm$

$dA = 2\pi r\ dr$

$dA = 2\pi(8)(0.02)$

$dA = 1.01\ mm^2$

---

8. Find the derivative of h with respect to u if $h = \pi^{2u}$

Ans: $2\pi^{2u} \ln \pi$

Solution:

$$\frac{d(a^u)}{x} = a^u \ln a \frac{du}{dx}$$

$$h = \pi^{2u}$$

$$\frac{dh}{du} = \pi^{2u} \ln\pi(2)$$

$$\frac{dh}{du} = 2\pi^{2u} \ln \pi$$

Alternate Solution:
Derivative using calculator:

Check from the choices:

---

9. If y = tan*h* x find dy/dx:
Ans: sec*h*^{2} x

Solution:

$$y = \tanh x$$

$$\frac{dy}{dx} = \mathrm{sech}^2\ x$$

Alternate Solution:
Derivative using calculator:

Check from the choices:

---

10. Find the derivative of y = x^x
Ans: x^x (1 + ln x)

Solution:

$$y = x^x$$

$$\ln y = x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \ln x\ (1)$$

$$\frac{dy}{dx} = y\ (1 + \ln x) = x^x(1 + \ln x)$$

Alternate Solution:
Derivative using calculator:

Check from the choices:

---

11. What is the derivative with respect to x of $(x+1)^3 - x^3$.

Ans: 6x + 3
Solution:

$$y = (x+1)^3 - x^3$$

$$y' = 3(x+1)^2 - 3(x)^2$$

$$y' = 3(x^2 + 2x + 1) - 3x^2$$

$$y' = 3x^2 + 6x = 3 - 3x^2$$

$$y' = 6x + 3$$

---

12. Find the slope of the ellipse $x^2 + 4y^2 - 10x - 16y + 5 = 0$ at the point where $y = 2 + 8^{0.5}$ and x = 7.
Ans: - 0.1768

Solution:

$$x^2 + 4y^2 - 10x - 16y + 5 = 0$$

$$2x + 8yy' - 10 - 16y' = 0$$

$$y = 2 + 8^{0.5}$$

$$y = 4.8284$$

$$x = 7$$

$$2(7) + 8(4.8284)\ y' - 10 - 16y' = 0$$

$$22.6274\ y' + 4 = 0$$

$$y' = -0.1768\ (\text{slope of ellipse})$$

---

13. Find the slope of the curve $y = 2(1+3x)^2$ at point $(0,3)$

Ans: 12
Solution:

$$y = 2(1+3x)^2$$

$$y' = 4(1+3x)(3)$$

$$y' = 12(1+3x) \text{ when } x = 0$$

$$y' = 12$$

---

14. If the slope of the curve $y^2 = 12x$ is equal to 1 at point (x,y) find the value of x and y.
Ans: x = 3, y = 6

Solution:

$$y^2 = 12x$$

$$2yy' = 12$$

$$y' = \frac{6}{y}$$

$$1 = \frac{6}{y}$$

$$y = 6$$

$$y^2 = 12x$$

$$(6)^2 = 12x$$

$$x = 3$$

---

15. Find the second derivative of $y = 2x + 3(4x + 2)^3$ when x=1.
Ans: 1728

Solution:

$$y = 2x + 3(4x+)^3$$

$$y' = 2 + 9(4x+2)^2(4)$$

$$y' = 2 + 36(4x+2)^2$$

$$y'' = 72(4x+2)(4)$$

$$y'' = 72(4)(6)$$

$$y'' = 1728$$

---

16. Find the second derivative of $y = x^{-2}$ when x = 2.
Ans: 0.375

Solution:

$$y = x^{-2}$$
$$y' = -2x^{-3}$$
$$y'' = 6x^{-4}$$
$$y'' = \frac{6}{x^4}$$
$$y'' = \frac{6}{(2)^4} = 0.375$$

17. Find the first derivative of $y = 2\,\text{Cos}\left(2 + x^2\right)$
Ans: - 4 x Sin (2 + x²)

Solution:

$$y = 2\,\text{Cos}\left(2 + x^2\right)$$
$$y' = 2(-\text{Sin})\left(2 + x^2\right)(2x)$$
$$y' = -4x\ \text{Sin}\left(2 + x^2\right)$$

---

18. Find the slope of the curve y= 6 $(4 + x)^{1/2}$ at point (0,12)
Ans: 1.5

Solution:

$$y = 6(4 + x)^{1/2}$$
$$y^2 = 36(4 + x)$$
$$2yy' = 36(1)$$
$$y' = \frac{36}{2(12)}$$
$$y' = 1.5$$

---

19. Find the point of inflection of the curve y= x^3 -3x^2 + 6.
Ans: 1,4

Solution:

$$y = x^3 - 3x^2 + 6$$
$$y' = 3x^2 - 6x$$
$$y'' = 6x - 6 = 0$$
$$6x - 6 = 0$$
$$x = 1$$
$$y = 1 - 3 + 6 = 4$$

Check :

when $x = 0 < 1$

$$y'' = 6(0) - 6 = -6$$

when $x = 2 > 1$

$$y'' = 6(2) - 6 = +6$$

therefore point of inflection is $(1,4)$

---

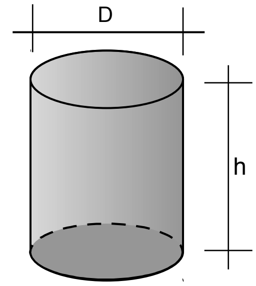
20. A cylindrical boiler is to have a volume of 1340 cu. ft. The cost of the metal sheets to make the boiler should be minimum. What should be its base diameter in feet?

Ans: 11.95

Solution:

$$V = \frac{\pi D^2}{4}h$$
$$1340 = \frac{\pi D^2}{4}h$$

To make cost be minimum,
the area should be minimum.



$$A = \frac{\pi D^2}{4}(2) + \pi D h$$
$$A = \frac{\pi D^2}{2} + \frac{\pi D(1340)(4)}{\pi D^2}$$
$$A = \frac{\pi D^2}{2} + \frac{1340(4)}{D}$$
$$\frac{dA}{dD} = \pi D - \frac{1340(4)}{D^2} = 0$$
$$\pi D = 1340(4)$$
$$D = 11.95$$

---

21. Find the two numbers whose sum is 12, if the product of one by the square of the other is to be maximum.
Ans: 4 and 8
Solution:

$$x = \text{one number}$$
$$12 - x = \text{other number}$$
$$P = x(12 - x)^2$$
$$\frac{dP}{dx} = x(2)(12 - x)(-1) + (12 - x)^2(1) = 0$$
$$2x(12 - x) = (12 - x)^2$$
$$2x = 12 - x$$
$$3x = 12$$
$$x = 4$$
$$12 - x = 8$$

The numbers are 4 and 8.

---

22. Find the two numbers whose sum is 20, if the product of one by the cube of the other is to be a maximum.
Ans: 5 and 15
Solution:

$x = \text{one number}$

$20 - x = \text{other number}$

$P = x(20 - x)^3$

$\frac{dP}{dx} = x(3)(20-x)^2(01) + (20-x)^3(1) = 0$

$3x(20-x)^2 = (20-x)^3$

$3x = 20 - x$

$4x = 20$

$x = 5$

$20 - 5 = 15$

23. A school sponsored trip will cost each student 15 pesos if not more than 150 students make the trip, however the costs per student will be reduced by 5 centavos for each student in excess of150. How many students should make the trip in order for the school to receive the largest group income?
Ans: 225

Solution:

$x = \text{additional student that will join the trip}$

$I = (150 + x)(15 - 0.05x)\ (\text{gross income})$

$\frac{dI}{dx} = (150 + x)(-0.05) + (15 - 0.05)(1) = 0$

$0.05(150 + x) = 15 - 0.05$

$75 + 0.05x = 15 - 0.05x$

$0.10x = 7.5$

$x = 75$

$\text{Total number of students} = 150 + 75$

$\text{Total number of students} = 225$

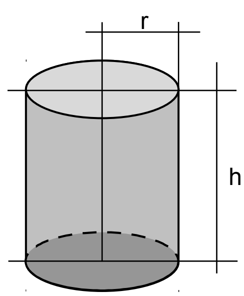
24. A closed cylindrical tank has a capacity of 576.56 m^3. Find the minimum surface area of the tank.
Ans: 383.40 m^3
Solution:

$V = \pi r^2$

$576.56 = \pi r^2 h$

$h = \frac{576.56}{\pi r^2}$

$\text{Surface Area}:$



$S = 2\pi r^2 + 2\pi rh$

$S = 2\pi r^2 + \frac{2\pi r(576.56)}{\pi r^2}$

$S = 2\pi r^2 + \frac{1153.12}{r}$

$\frac{dS}{dr} = 4\pi r - \frac{1153.12}{r^2} = 0$

$4\pi r^3 = 1153.12$

$r = 4.51$

$h = \frac{576.56}{\pi(4.51)^2}$

$h = 9.02$

$S = 2\pi r^2 + 2\pi rh$

$S = 2\pi(4.51)^2 + 2\pi(4.51)(9.02)$

$S = 383.40\ m^2.$

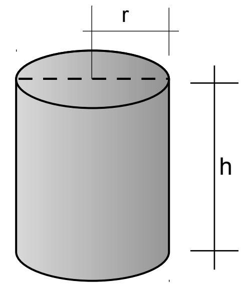
25. A closed cylindrical tank has a capacity of 16π cu.m. Determine the radius and height of the tank that requires minimum amount of material used.
Ans: 2 m., 4 m.

Solution:

$V = \pi r^2 h$

$16\pi = \pi r^2 h$

$h = \frac{16}{r^2}$



$A = 2\pi r^2 + 2\pi rh$

$A = 2\pi r^2 + \frac{2\pi r(16)}{r^2}$

$A = 2\pi r^2 + \frac{32\pi}{r}$

$\frac{dA}{dr} = 4\pi r - \frac{32\pi}{r^2} = 0$

$4\pi r^3 = 32\pi$

$r^3 = \frac{32}{4}$

$r = 2m.$

$h = \frac{16}{4}$

$h = 4\ m.$

26. The stiffness of a rectangular beam is proportional to the breadth and the cube of the depth. Find the shape of the stiffest beam that can be cut from a log of given size.
Ans: depth = $\sqrt{3}$ breadth

Solution:

$S \alpha bd^3$

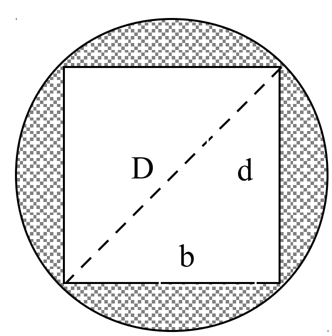
$S = Kbd^3$

$D^2 = d^2 + b^2$

$b = \sqrt{D^2 - d^2}$

$S = K\sqrt{D^2 - d^2}\left(d^3\right)$

$\frac{ds}{dd} = K\sqrt{D^2 - d^2}\left(3d^2\right) + \frac{Kd^3(-2d)}{2\sqrt{D^2 - d^2}} = 0$



$3\left(D^2 - d^2\right)d^2 = d^4$

$3\left(D^2 - d^2\right) = d^2$

$3D^2 - 3d^2 = d^2$

$4d^2 = 3D^2$

$4d^2 = 3\left(d^2 + b^2\right)$

$4d^2 = 3d^2 + 3b^2$

$d^2 = 3b^2$

$d = \sqrt{3}\,b$

27. What is the maximum length of the perimeter if the hypotenuse of a right triangle is 5 m. long?
Ans: 12.08 m.

Solution:

$x^2 + y^2 = 25$

$y = \sqrt{25 - x^2}$

$P = x + y + 5$

$P = x + \sqrt{25 - x^2} + 5$

$\frac{dP}{dx} = 1 - \frac{2x}{2\sqrt{25 - x^2}} = 0$

$\sqrt{25 - x^2} = x$

$25 = 2x^2$

$x^2 = 12.5$

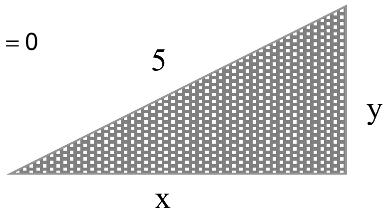
$x = 3.54$

$y = \sqrt{25 - x^2}$

$y = 3.54$

$P = 3.54 + 3.54 + 5$

$P = 12.08$ m.



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28. If the sum of the two numbers is 4, find the minimum value of the sum of their cubes.
Ans: 16
Solution:

$x + y = 4$

$S = x^3 + y^3$

$S = x^3 + (4 - x)^3$

$\frac{dS}{dx} = 3x^2 + 3(-x^2)(-1) = 0$

$3x^2 = 3(4 - x)^2$

$x = 4 - x$

$x = 2 \quad ; \quad y = 2$

$S = (2)^3 + (2)^3$

$S = 16$

---

29. The highway department is planning to build a picnic area for motorist along a major highway. It is to be a rectangle with an area of 5000 sq. m to be fenced off on the three sides not adjacent to the highway. What is the least amount of fencing that will be needed to complete the job?
Ans: 200 m.

Solution:

$A = xy$

$5000 = xy$

$y = \frac{500}{x}$

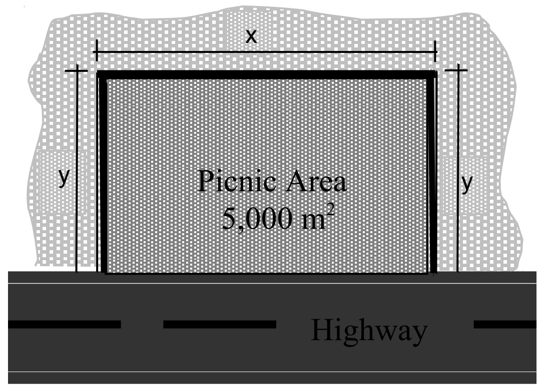
$P = 2y + x = \frac{2(5000)}{x} + x$

$\frac{dP}{dx} = -\frac{10000}{x^2} + 1 = 0$

$x = 100$

$y = \frac{5000}{100} = 50$

Least amount of fencing = 2y + x
Least amount of fencing = 2(50) + 100
Least amount of fencing = 200 m.



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30. A student club on a college campus charges annual membership dues of P10, less 5 centavos for each member over 60. How many members would give the club the most revenue from annual dues?
Ans: 130 members
Solution:

x = no. of members

$10 - 0.05(x - 60)$ = discounted price if there are more then 60 members

$x - 60$ = excess no. of members

Total revenue $= x\left[10 - 0.05(x - 60)\right]$

$R = 10x - 0.05x^2 + 3x$

$R = 13x - 0.05x^2$

$\frac{dR}{dx} = 13 - 0.10x = 0$

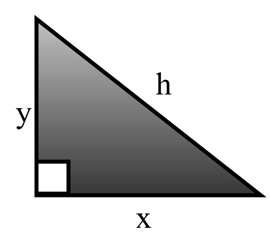
$x = 130$ members

---

31. If the hypotenuse of a right triangle is know, what is the relation of the base and the altitude of the right triangle when its area is maximum.
Ans: Altitude = base

Solution:
For maximum area of a triangle with known hypotenuse, the triangle should be isosceles:

From the figure: $x = y$



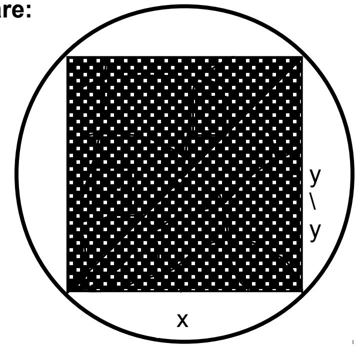
32. What is the area in the sq. m. of the rectangle of maximum perimeter inscribed in a circle having a diameter of 20 m.
Ans: 200

Solution:
For maximum rectangle inscribed in a circle, the rectangle should be a square:

From the figure:
$x = y$

By Pythagorean Theorem:

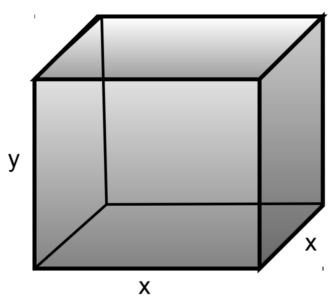
$$x^2 + x^2 = 20^2$$
$$2x^2 = 400$$
$$x^2 = 200$$



33. The sum of the lengths of all the edges of a closed rectangular box is equal to 6m. If the top and the bottom are equal squares. What height in meters will give the maximum volume?
Ans: $\frac{1}{2}$

Solution:

$$4x(2) + 4y = 6$$
$$8x + 4y = 6$$
$$4x + 2y = 3$$
$$y = \frac{3-4x}{2}$$
$$V = x^2 y$$
$$V = x^2\left(\frac{3-4x}{2}\right)$$
$$\frac{dV}{dx} = \frac{1}{2}\left[x^2(-4) + (3-4x)(2x)\right] = 0$$
$$4x^2 = 6x - 8x^2$$
$$12x^2 = 6x$$
$$12x = 6$$
$$x = \frac{1}{2}$$
$$y = \frac{3-4\left(\frac{1}{2}\right)}{2}$$
$$y = \frac{3-2}{2}$$
$$y = \frac{1}{2}$$

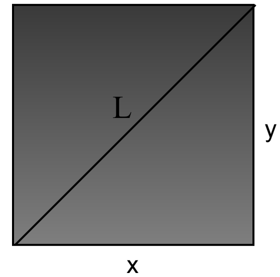


Therefore the height $= \frac{1}{2}$m.

34. What is the shape of the rectangle of given area that has the longest diagonal?
Ans: length = width

Solution:

$$A = xy$$
$$0 = xy' + y(1)$$
$$y' = -\frac{y}{x}$$
$$L^2 = x^2 + y^2$$
$$2LL' = 2x + 2yy' = 0$$
$$y' = -\frac{x}{y}$$
$$\frac{-y}{x} = \frac{-x}{y}$$
$$x = y$$



The rectangle should be a square

35. Find two numbers whose sum is 20 and whose product is maximum.
Ans: 10, 10

Solution:

$x =$ one number

$20 - x =$ other number

$$P = x(20 - x)$$
$$\frac{dP}{dx} = x(-1) + (20-x)(1) = 0$$
$$x = 20 - x$$
$$2x = 20$$

$x = 10$ (one number)

$20 - x = 10$ (other number)

36. Find two numbers whose sum is 36 if the product of one by the square of the other is a maximum.
Ans: 12,24

Solution:

$x =$ one number

$36 - x =$ other number

$$P = x(36-x)^2$$
$$\frac{dP}{dx} = x(2)(36-x)(-1) + (36-x)^2(1) = 0$$
$$2x(36-x) = (36-x)^2$$
$$2x = 36 - x$$
$$3x = 36$$
$$x = 12$$
$$36 - x = 24$$

37. The selling price of a certain commodity is 100 – 0.02x pesos when "x" is the number of commodity produced per day. If the cost of producing and selling "x" commodity is 15000 + 40x pesos per day, how many commodities should be produced and sold everyday in order to maximize the profit?
Ans: 1500

Solution:

$$\text{Profit} = \text{selling price} - \text{production cost}$$
$$P = x(100 - 0.02x) - (15000 + 40x)$$
$$\frac{dP}{dx} = x(-0.02) + (100 - 0.02x)(1) - 40 = 0$$
$$-0.02x + 100 - 0.02x - 40 = 0$$
$$0.04x = 60$$
$$x = 1500 \text{ number of commodity produced per pay}$$

---

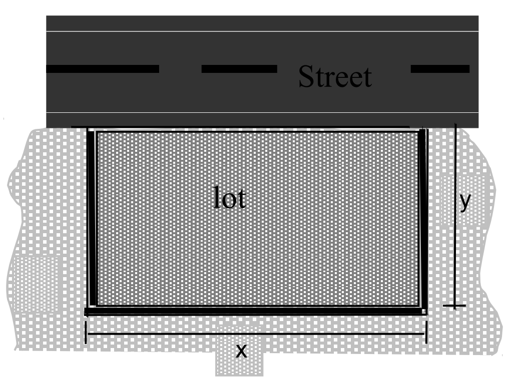
38. The cost of a product is a function of the quantity x of the product. If C (x) $=x^2-2000x + 100$, find the quantity x for w/c the cost is minimum.
Ans: 1000

Solution:
$$C = x^2 - 2000x + 100$$
$$C = 2x - 2000 = 0$$
$$x = 1000$$

---

39. A buyer is to take a plot of land fronting a street, the plot is to be a rectangular and three times its frontage added to twice its depth is to be 96 meters. What is the greatest number of square meters he may take?
Ans: 384 sq.m

Solution:
$$A = xy$$
$$3x + 2y = 96$$
$$y = \frac{96 - 3x}{2}$$
$$A = xy$$
$$A = \frac{x(96 - 3x)}{2}$$
$$\frac{dA}{dx} = \frac{1}{2}\left[x(-3) + (96 - 3x)(1)\right] = 0$$
$$3x = 96 - 3x$$
$$x = \frac{96}{6} = 16$$
$$y = \frac{96 - 3(16)}{2} = 24$$
$$A = xy$$
$$A = 16(24)$$
$$A = 384 \text{ sq.m.}$$



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40. The following statistics of a manufacturing company shows the corresponding values for manufacturing x" units.
Producing cost=60 x + 10000 pesos
Selling price/unit = 200 – 0.02 x pesos
How many units must be produced for max. profit.
Ans: 3500

Solution:
$$\text{Profit} = \text{Sales} - \text{Production cost}$$
$$P = (200 - 0.02x)\,x - (60x + 10000)$$
$$\frac{dP}{dx} = (200 - 0.02x)(1) + x(-0.02) - 60 = 0$$
$$140 = 0.04x$$
$$x = 3500 \text{ units}$$

---

41. The cost per unit of production is expressed as $(4 + 3x)$ and the selling price on the competitive market is P100 per unit. What maximum daily profit that the company can expect of this product?
Ans: P768

Solution:
$$\text{Profit} = \text{Income} - \text{expenses or cost}$$
$$P = 100x - (4 + 3x)\,x$$
$$\frac{dP}{dx} = 100 - \left[(4 + 3x)(1) + x(3)\right] = 0$$
$$100 = 4 + 3x + 3x$$
$$6x = 96$$
$$x = 16$$
$$\text{Profit} = 100(16) - \left[4 + 3(16)\right]16$$
$$\text{Profit} = P768$$

---

42. A certain unit produced by the company can be sold for 400 -0.02x pesos where "x" is the number of units manufactured. What would be the corresponding price per unit in order to have a maximum revenue?
Ans: P200

Solution:
$$\text{Revenue} = (400 - 0.02x)\,x$$
$$R = 400x - 0.02x^2$$
$$\frac{dR}{dx} = 400 - 0.04x = 0$$
$$x = 1000 \text{ units}$$
$$\text{Unit price} = 400 - 0.02x$$
$$\text{Unit price} = 400 - 0.02(10000)$$
$$\text{Unit price} = P200$$

43. Given the cost equation of a certain product as follows $C=50t^2 - 200t + 10000$ where t is in years. Find the maximum cost from the year 1995 to 2002.
Ans: P9,800

Solution:

$$C = 50t^2 - 200t + 10000$$
$$\frac{dC}{dt} = 100t - 200 = 0$$
$$t = \frac{200}{100}$$
$$t = 2 \text{ yrs.}$$

Max. cost will occur in year 1997.
Max. cost = $50(2)^2 - 200(2) + 10000$
Max. cost = P9,800

44. The demand "x" for a product is x=10000 -100P where "P" is the market price in pesos per unit. The expenditure for the two product is E =Px. What market price will the expenditure be the greatest?
Ans: 50
Solution:

$$E = Px$$
$$E = P(10000 - 100P)$$
$$E' = P(-100) + (10000 - 100P)(1) = 0$$
$$100P = 10000 - 100P$$
$$P = 50$$

45. Analysis of daily output of a factory shows that the hourly number of units "y" produced after "t" hours of production is $y=70t + \frac{r^2}{2} - t^3$. After how many hours will the hourly number of units be maximized?
Ans: 5
Solution:

$$y = 70t + \frac{t^2}{2} - t^3$$
$$y' = 70 + t - 3t^2 = 0$$
$$3t^2 - t - 70 = 0$$
$$(3t + 14)(t - 5) = 0$$
$$t = 5 \text{ hours}$$

46. A car manufacturer estimates that the cost of production of "x" cars of a certain model is $C=20x-0.01x^2-800$. How many cars should be produced for minimum cost?
Ans: 1000
Solution:

$$C = 20x - 0.01x^2 - 800$$
$$\frac{dC}{dx} = 20 - 0.02x = 0$$
$$x = 1000 \text{ cars}$$

47. If the total profit in thousand pesos for a product is given by $P = 20\sqrt{x+1} - 2x$, what is the marginal profit at a production level of 15 units, where P is the profit and "x" the no. of units produced.
Ans: P500
Solution:

$$P = 20\sqrt{x+1} - 2x$$
$$P' = 10(x+1)^{-1/2} - 2$$
$$P' = 10(15+1)^{-1/2} - 2$$
$$P' = 0.50$$
$$P' = P500 \text{ (Marginal profit)}$$

48. A firm in competitive market sells its product for P200 per unit. The cost per unit (per month) is 80 + x, where x represents the number of units sold per month. Find the marginal profit for a production of 40 units.
Ans: P40

Solution:

$$P = 200x - (80 + x)x$$
$$P = 200x - 80x + x^2$$
$$P = 120x - x^2$$
$$P' = 120 - 2x$$
$$P' = 120 - 2(240)$$
$$P' = P40 \text{ (marginal profit)}$$

49. A time study showed that on average, the productivity of a worker after "t" hours on the job can be modeled by the expression $P=27 + 6t - t^3$ where P is the number of units produced per hour. What is the maximum productivity expected?
Ans: 36

Solution:

$$P = 27 + 6t - t^3$$
$$P' = 6 - 3t^2 = 0$$
$$t = 3$$
$$P = 27 + 6(3) - (3)^2$$
$$P = 36 \text{ (max. productivity)}$$

50. The number of parts produced per hour by a worker is given by $P = 4 + 3t^2 - t^3$ where "t" is the number of hours on the job without a break. If a worker starts at 8:00 A.M., when will she be at maximum production during the morning?
Ans: 10:00 A.M.
Solution:

$$P = 4 + 3t^2 - t^3$$
$$P' = 6t - 3t^2 = 0$$
$$t = 2 \text{ hours.}$$

Max. production will be at 8 + 2 = 10:00 A.M.

51. Find the abscissa on the curve $x^2 = 2y$ which is nearest to a point (4,1).
Ans: 2

Solution:

$$d=\sqrt{(x-4)^2+(y-1)^2}$$

$$x^2=2y$$

$$y=\frac{x^2}{2}$$

$$d=\sqrt{(x-4)^2+\left(\frac{x^2}{2}-1\right)^2}$$

$$d'=\frac{2(x-4)+2\left(\frac{x^2}{2}-1\right)\left(\frac{2x}{2}\right)}{2\sqrt{(x-4)^2+\left(\frac{x^2}{2}-1\right)^2}}=0$$

$$x-4+\left(\frac{x^2}{2}-1\right)x=0$$

$$2x-8+x^3-2x=0$$

$$x^3=8$$

$$x=2$$

52. Find the equation of the tangent line to the curve $y=x^3-3x^2+5x$ that has the least slope.
Ans: 2x – y + 1 = 0

Solution:

$$y=x^3-3x^2+5x$$

$$y'=3x^2-6x+5 \text{ slope of pt. of tangency}$$

$$m=3x^2-6x+5$$

$$\frac{dm}{dx}=6x-6=0$$

$$x=1$$

$$y=1-3+5$$

$$y=3$$

$$m=3(1)^2-6(1)+5$$

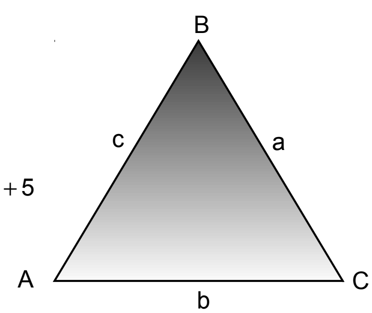
$$m=2$$

$$m=\frac{y-y_1}{x-x_1}$$

$$2=\frac{y-3}{x-1}$$

$$2x-2=y-3$$

$$2x-y+1=0$$



53. The cost C of a product is a function of the quantity x of the product: $C(x)=x^2-4000x+50$. Find the quantity for which the cost is minimum.
Ans: 2000

Solution:

$$C=x^2-4000x+50$$

$$\frac{dc}{dx}=2x-4000=0$$

$$x=2000$$

54. Divide 60 into 3 parts so that the product of the three parts will be a maximum, find the product.
Ans: 8000

Solution:

$$P=20(20)(20)$$

$$P=8000$$

55. Find the radius of the circle inscribe in a triangle having a maximum area of 173.205 cm^2.
Ans: 5.77 cm.

Solution:

For max. area of a triangle, the triangle is an equilateral triangle.

$$173.205=\frac{x^2\text{Sin }60^\circ}{2}$$

$$x=20\text{ cm.}$$

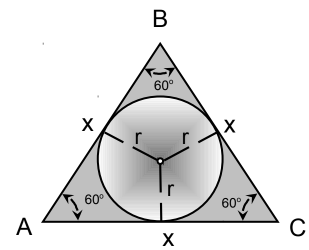
$$A=rS$$

$$S=\frac{x+x+x}{2}$$

$$S=30$$

$$173.205=r(30)$$

$$r=5.77\text{ cm.}$$



56. Find the perimeter of a triangle having a max. area, that is circumscribing as circle of radius 8 cm.
Ans: 83.13 cm

Solution:

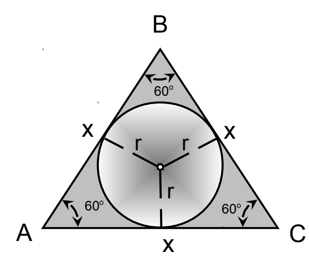
$$A=r\,S$$

$$\frac{x^2\text{Sin}60^o}{2}=\frac{8(3x)}{2}$$

$$x=27.71$$

$$\text{Perimeter}=27.71(3)$$

$$\text{Perimeter}=83.13\text{cm.}$$

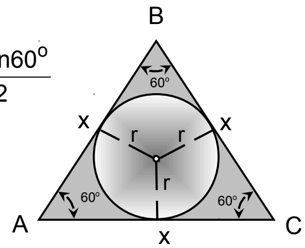


57. The area of a circle inscribe in a triangle is equal to 113.10 cm^2. Find the maximum area of the triangle.
Ans: 186.98 cm²
Solution:

$$A_c = \pi r^2$$

$$113.10 = \pi r^2$$

$$r = 6 \text{ cm}$$



$$\text{Area of triangle} = \frac{x^2 \text{Sin}60^o}{2}$$

$$\frac{x^2 \text{Sin}60^o}{2} = r\ S$$

$$S = \frac{3x}{2}$$

$$\frac{x^2 \text{Sin}60^o}{2} = \frac{6(3x)}{2}$$

$$x = 20.78 \text{ cm.}$$

$$A_{max} = \frac{(20.78)^2 \text{ Sin } 60^o}{2}$$

$$A_{max} = 186.98 \text{ cm}^2$$

---

58. The sides of an equilateral triangle area increasing at the rate of 10 m/s. What is the length of the sides at the instant when the area is increasing 100 sq. m/sec.?

Ans: $\frac{20}{\sqrt{3}}$

Solution:

$$h = x\text{Sin}60^o$$

$$h = \frac{x\sqrt{3}}{2}$$

$$A = \frac{hx}{2}$$

$$A = \frac{x\sqrt{3}x}{4}$$

$$A = \frac{x^2\sqrt{3}}{4}$$

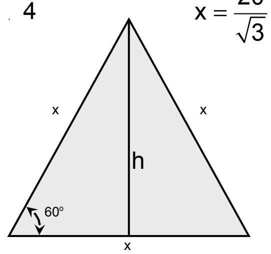
$$\frac{dx}{dt} = 10 \text{m/sec.}$$

$$\frac{dx}{dt} = 100 \text{sq.m./sec}$$

$$\frac{dA}{dt} = 2x\frac{dx}{dt}\frac{\sqrt{3}}{4}$$

$$100 = \frac{2}{4}x(10)\sqrt{3}$$

$$x = \frac{20}{\sqrt{3}} \text{meters}$$



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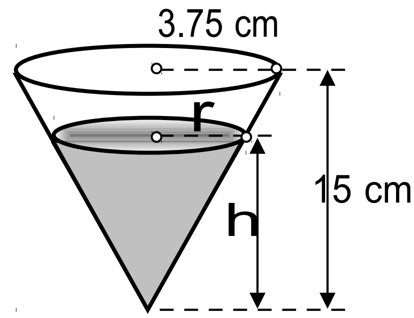
59. Water is flowing into a conical vessel 15 cm. deep and having a radius of 3.75 cm. across the top. If the rate at which water is rising is 2 cm/sec. How fast is the water flowing into the conical vessel when the depth of water is 4 cm?
Ans: 6.28 m³/min.

Solution:



$$V = \frac{\pi r^2 h}{3}$$

$$\frac{r}{h} = \frac{3.75}{15}$$

$$r = \frac{h}{4}$$

$$V = \frac{\pi r^2 h}{3}$$

$$V = \frac{\pi (h^2) h}{3(16)}$$

$$V = \frac{\pi h^3}{48}$$

$$\frac{dV}{dt} = \frac{\pi}{48}(3)h^2\frac{dh}{dt}$$

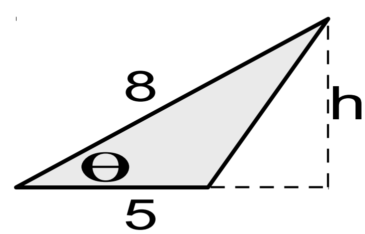
$$\frac{dV}{dt} = \frac{\pi}{16}(4)^2(2)$$

$$\frac{dV}{dt} = 6.28 \text{ m}^3/\text{min.}$$

---

60. The two adjacent sides of a triangle are 5 and 8 meters respectively. If the included angle is changing at the rate of 2 rad/sec., at what rate is the area of the triangle changing if the included angle is 60°?
Ans: 20 sq. m. /sec

Solution:



$$A = \frac{h5}{2}$$

$$h = 8\ \text{Sin}\theta$$

$$A = \frac{8\ \text{Sin}\theta(5)}{2}$$

$$\frac{dA}{dt} = 20\ \text{Cos}\theta\frac{d\theta}{dt}$$

$$\frac{dA}{dt} = 20\ \text{Cos}60^o(2)$$

$$\frac{dA}{dt} = 20 \text{ sq.m. /sec.}$$

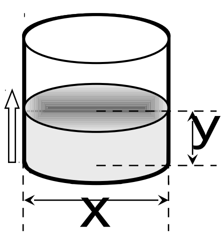
61. Find the point in the parabola $y^2 = 4x$ at which the rate of change of the ordinate and abscissa are equal.
Ans: (1,2)

Solution:

$$y^2 = 4x$$
$$2y\frac{dy}{dt} = 4\frac{dx}{dt}$$
$$\text{but; } \frac{dy}{dt} = \frac{dx}{dt}$$
$$2y = 4$$
$$y = 2$$
$$(2)^2 = 4x$$
$$x = 1$$
The point is $(1,2)$

62. Water flows into a vertical cylindrical tank, at the rate of 1/5 cu. ft./sec. The water surface is rising at the rate of 0.425 ft. per minute. What is the diameter of the tank?
Ans: 6 ft.

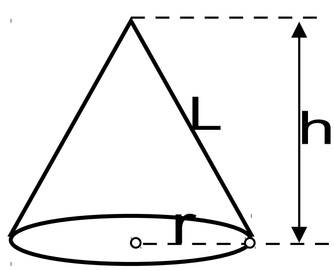
Solution:



$$V = \frac{\pi}{4}x^2 y$$
$$\frac{dv}{dt} = \frac{\pi}{4}x^2\frac{dy}{dt}$$
$$\frac{1}{5} = \frac{\pi}{4}x^2\frac{0.425}{60}$$
$$x^2 = \frac{4(60)}{5(\pi)(0.425)} = \frac{4(12)}{\pi(0.425)}$$
$$x^2 = 36$$
$$x = 6\text{ft.}$$

63. The base radius of a cone is changing at a rate of 2 cm/sec. Find the rate of change of its lateral area when it has an altitude of 4 cm. and a radius of 3 cm.
Ans: 10 $\pi\ cm^2/sec.$

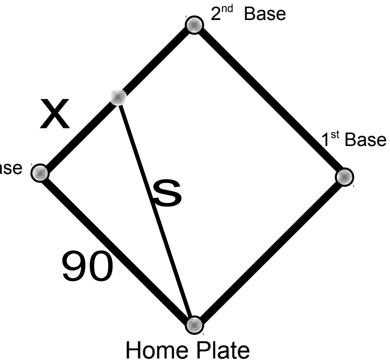
Solution:



$$A = \pi rL$$
$$L^2 = h^2 + r^2$$
$$L^2 = (4)^2 + (3)^2$$
$$L = 5$$
$$\frac{dA}{dt} = \pi\frac{dr}{dt}L$$
$$\frac{dA}{dt} = \pi(2)(5)$$
$$\frac{dA}{dt} = 10\pi cm^2/sec.$$

64. A baseball diamond has the shape of a square with sides 90 meters along. A player 30 m. from the third base and 60 m. from the 2nd base is running at a speed of 28 m/sec. At what rate is the players distance from the home plate changing?
Ans: 8.85 m/sec.

Solution:



$$S^2 = 30^2 + (90)^2$$
$$S = \sqrt{x^2 + 8100}$$
$$\frac{dS}{dt} = \frac{2x\frac{dx}{dt}}{2\sqrt{x^2+8100}}$$
$$\text{when } x = 30$$
$$\frac{dx}{dt} = -28\text{m/sec.}$$
$$\frac{dS}{dt} = \frac{30(-28)}{\sqrt{(30)^2+8100}}$$
$$\frac{dS}{dt} = -8.85\text{ m/s}$$

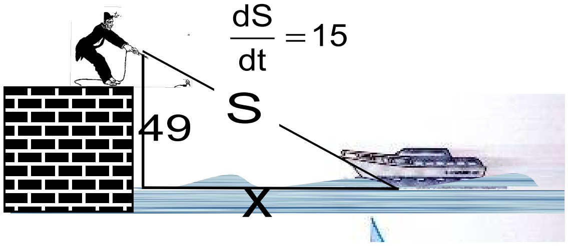
65. A bridge is 10m. above a canal. A motor boat going 3 m/sec. passes under the center of the bridge at the same instant that a woman walking 2 m/sec. reaches that point. How rapidly are they separating 3 sec. later?
Ans: 2.65

Solution:

$$S^2 = x^2 + 100 + y^2$$
$$S = \sqrt{x^2 + y^2 + 100}$$
$$\frac{dS}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2\sqrt{x^2+y^2+100}}$$
$$x = 2(3) = 6$$
$$y = 3(3) = 9$$
$$\frac{dS}{dt} = \frac{6(2)+9(3)}{\sqrt{(36+81+100)}}$$
$$\frac{dS}{dt} = 2.56\text{m/sec.}$$

66. A launch whose deck is 7m. below the level of a wharf is being pulled toward the wharf by a rope attached to a ring on the deck. If a winch pulls in the rope at the rate of 15 m/min., how fast is the launch moving through the water when there are 25 m. of rope out?
Ans: -15.625

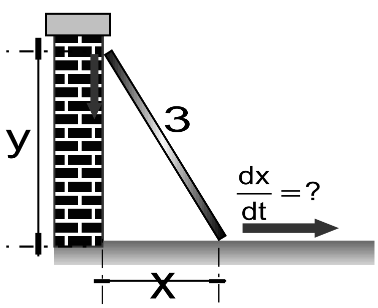
Solution:



$$S^2 = 49 + x^2$$
$$S = \sqrt{x^2+49}$$
when $S = 25$
$$(25)^2 = 49 + x^2$$
$$x = 24m.$$
$$\frac{dS}{dt} = \frac{2x\ dx/dt}{2\sqrt{x^2+49}}$$
$$-15 = \frac{24\ dx/dt}{\sqrt{(24)^2+49}}$$
$$\frac{dx}{dt} = -15.625\ m/min.$$

67. A 3 m. steel pipes is leaning against a vertical wall and the other end is on the horizontal floor. If the lower end slides away from the wall at 2 cm/sec., how fast is the other end sliding down the wall when the lower end is 2m. from the wall?
Ans: -1.79 cm/s

Solution:



$$x^2 + y^2 = 9$$
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$
when $x = 2$
$$4 + y^2 = 9$$
$$y^2 = 5$$
$$y = \sqrt{5}$$
$$2(2)(0.02) + (\sqrt{5})\frac{dy}{dt} = 0$$
$$\frac{dy}{dt} = -0.0179\ m/s$$
$$\frac{dy}{dt} = -1.79\ cm/s$$

68. Two sides of a triangle are 30 cm. and 40 cm. respectively. How fast is the area of the triangle increasing if the angle between the sides is 60° and is increasing at the rate of 4 degrees/sec.
Ans: 20.94

Solution:
$$A = \frac{30(40)}{2}\text{Sin}\theta$$
$$A = 600\ \text{Sin}\theta$$
$$\frac{dA}{dt} = 600\ \text{Cos}\theta\frac{d\theta}{dt}$$
$$\frac{dA}{dt} = 600\ \text{Cos}\ 60^o\frac{(4)\pi}{180}$$
$$\frac{dA}{dt} = 20.94\ sq.m./sec.$$

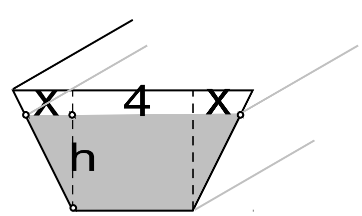
69. A particle moves in a plane according to the parametric equation of motions: $x = t^2$, $y=t^3$. Find the magnitude of the acceleration $t = \frac{2}{3}$.
Ans: 4.47
(ANSWER & SOLUTION FROM THE SOURCE)

Solution:
$$x = t^2$$
$$\frac{dx}{dt} = -2t$$
$$Vx = -2t(\text{velocity})$$
$$a_x = -2(\text{acceleration})$$
$$y = t^3$$
$$\frac{dy}{dt} = 3t^2$$
$$v_y = 3t^2$$
$$a_y = 6t$$
$$a_y = 6\left(\frac{2}{3}\right)$$
$$a_y = 4$$
Thus,
$$a = \sqrt{(-2)^2+(4)^2}$$
$$a = 4.47$$

70. A horizontal trough is 16 m. long and its ends are isosceles trapezoids with an altitude of 4m., a lower base of 4m. and an upper base of 6 m. If the water level is decreasing at the rate of 25 cm/min. when the water is 3m. deep, at what rate is water being drawn from the trough?

Ans. 22 m^3/min.

Solution:



$$\frac{x}{h}=\frac{1}{4}$$
$$x=\frac{h}{4}$$
$$dV=(4+2x)(16)\,dh$$
$$dV=\left(4+\frac{h}{2}\right)16\,dh$$
$$\frac{dV}{dt}=\left(4+\frac{3}{2}\right)(16)(0.25)$$
$$\frac{dV}{dt}=22m^3/\min.$$

71. A point moves on the curve $y=x^2$. How fast is "y" changing when x=2 and x is decreasing at a rate 3?

Ans. 12

Solution:

$$y=x^2 \qquad x=-2 \qquad dx=-3$$
$$dy=2xdx$$
$$dy=2(-2)(-3)$$
$$dy=+12$$

72. A particle moves along a path whose parametric equations are $x=t^3$ and $y=2t^2$. What is the acceleration when t=3 sec.

Ans. 18.44 m/sec^2

Solution:

$$x=t^3$$
$$\frac{dx}{dt}=3t^2$$
$$V_x=3t^2$$
$$\frac{d(V_x)}{dt}=6t$$
$$a_x=6t$$
$$a_x=6(3)$$
$$a_x=18m/sec^2$$
$$y=2t^2$$
$$\frac{dy}{dt}=4t$$
$$V_y=4t$$
$$\frac{d(V_y)}{dt}=4$$
$$a_y=4$$
$$a=\sqrt{ax^2+ay^2}$$
$$a=\sqrt{(18)^2+(4)^2}$$
$$a=18.44m/sec^2$$

73. A light is at the top of a pole 80 ft. high. A ball is dropped at the same height from a point 20 ft. from the light. Assuming that the ball falls according to $S=16t^2$, how fast is the shadow of the ball mobbing along the ground 1 second later?

Ans. 200 ft/sec.

Solution:

$$\frac{20}{S}=\frac{20+x}{80}$$
when t = 1sec.
$$S=16t^2$$
$$\frac{dS}{dt}=16(2)t$$
$$\frac{dS}{dt}=32\,ft/sec.$$
when t = 1
$$S=16$$
$$16=\frac{1600}{20+x}$$
$$20+x=100$$
$$x=80$$
$$S=\frac{1600}{20+x}$$
$$\frac{dS}{dt}=\frac{-1600}{(20+x^2)}\frac{dx}{dt}$$
$$32=\frac{-1600}{(100)^2}\frac{dx}{dt}$$
$$\frac{dx}{dt}=-200\,ft/sec.$$

74. A point moves on a parabola $y^2=4x$ in such a way that the rate of change of the abscissa is always 2 units per minute. How fast is the ordinate changing when the ordinate is 5.

Ans. 0.8 units per minute

Solution:

$$y^2=4x$$
$$2y\frac{dy}{dt}=4\frac{dx}{dt}$$
$$\frac{dy}{dt}=\frac{2\frac{dx}{dt}}{y}$$
when y = 5
$$\frac{dy}{dt}=\frac{2(2)}{5}$$
$$\frac{dy}{dt}=0.8\text{ units per minute}$$

75. Water is poured at the rate of $8ft^3$/min into a conical shaped tank, 20 ft. deep and 10 ft. diamtere at the top. If the tank has aleak in the bottom and the water level is rising at the rate of 1 inch/min., when te water is 16 ft. deep, how fast is the water leaking?

Ans. 3.81 ft^3/min.

Solution:

$$V = \frac{\pi r^2 h}{3}$$
$$\frac{r}{h} = \frac{5}{20}$$
$$r = \frac{1}{4}h$$
$$V = \frac{\pi r^2 h}{3}$$
$$V = \frac{\pi r^2 h}{3(16)}$$
$$\frac{dV}{dt} = \frac{3\pi h^2}{3(16)}\frac{dh}{dt}$$
$$\frac{dV}{dt} = \frac{\pi(16)^2}{16}\left(\frac{1}{12}\right)$$
$$\frac{dV}{dt} = 4.19\,ft^3/min.$$
$$Q_1 - Q_2 = \frac{dV}{dt}$$
$$8 - Q_2 = 4.19$$
$$Q_2 = 3.81 ft^3/min.$$

76. The sides of an equilateral triangle is increasing at rate of 10 cm/min. What is the length of the sides if the area is increasing at the rate of 69.82 cm^2/min.

Ans. 8 cm.

Solution:

$$A = \frac{x^2 \text{Sin}\,60°}{2}$$
$$A = 0.433x^2$$
$$dA = 0.433(2)\,x\,dx$$
$$69.28 = 0.866x(10)$$
$$x = 8\,cm.$$

77. Find the point in parabola y^2=4x at which the rate of change of the ordinate and abscissa are equal.

Ans. 1,2
Solution:

$$y^2 = 4x$$
$$2y\frac{dy}{dx} = 4\frac{dx}{dt}$$
$$\frac{dy}{dt} = \frac{dx}{dt}$$
$$2y = 4$$
$$y = 2$$
$$\text{when } y = 2$$
$$(2)^2 = 4x$$
$$x = 1$$
$$\text{The point is } (1,2)$$

78. What is pouring into a swimming pool. After t hours, there are t+ $\sqrt{t}$ gallons in the pool. AT what rate is the water pouring into the pool when t-=9 hours?

Ans. 7/6 gph
Solution:

$$V = t + \sqrt{t}\ \text{(vol. of water in the pool)}$$
$$\frac{dV}{dt} = \text{rate at which is pouring into the pool}$$
$$\frac{dV}{dt} = 1 + \frac{1}{2\sqrt{t}}$$
$$\frac{dV}{dt} = 1 + \frac{1}{2(\sqrt{9})}$$
$$\frac{dV}{dt} = 1 + \frac{1}{6}$$
$$\frac{dV}{dt} = \frac{7}{6}\text{gph}$$

79. Determine the velocity of progress with the given equation

$$D = 20t + \frac{5}{t+1} \quad \text{when } t = 4\,\text{sec.}$$

Ans. 19.8 m/s
Solution:

$$D = 20t + \frac{5}{(t+1)}$$
$$D' = 20 - \frac{5}{(t+1)^2}$$
$$V = 20 - \frac{5}{(5)^2}$$
$$V = 19.8\,m/s$$

80. A point on the rim of a flywheel of radius 5 cm., has a vertical velocity of 50 cm.sec. at a point P, 4 cm. above the x-axis. What is the angular velocity of the wheel?

Ans. 16,67 rad/sec.
Solution:

when $y = 4$

$$\text{Sin}\,\theta = \frac{4}{5}$$

$$\text{Cos}\,\theta = \frac{3}{5}$$

$$x = 5\,\text{Cos}\,\theta$$

$$y = 5\,\text{Sin}\,\theta$$

$$\frac{dy}{dt} = 5\,\text{Cos}\,\theta\frac{d\theta}{dt}$$

$$50 = 5\left(\frac{3}{5}\right)\frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = 16.67\,\text{rad/sec}\,(\text{angular velocity of the wheel})$$

---

81. What is the appropriate total area bounded by the curve y=Sin x and y=0 over the interval of $\le x, \le 2\pi$ (x in radians)

Ans. 4

Solution:

$$A = 2\int_0^{\pi} y\,dx$$

$$A = \int_0^{\pi} \text{Sin}\,x\,dx$$

$$A = 2\left[-\text{Cos}\,x\right]_0^{\pi}$$

$$A = -2\left(\text{Cos}\pi - \text{Cos}\theta\right)$$

$$A = -2\left[-1-1\right]$$

$$A = -2(-2)$$

$$A = 4$$

---

82. Find the area bounded by the curve $y=8-x^3$ and the x-axis.

Ans. 12 sq. units

Solution:

$$A = \int_0^2 y\,dx$$

$$A = \int_0^2 \left(8 - x^3\right)dx$$

$$A = \left[8x - \frac{x^4}{4}\right]_0^2 = \left[8(2) - \frac{(2)^4}{4}\right]$$

$$A = 16 - \frac{16}{4}$$

$$A = 16 - 4$$

$$A = 12\,\text{square units}$$

---

83. Find the distance of the centroid from the y-axis of the area bounded by the curve $x^2=16y$, the line x=12 and the x-axis.

Ans. 9

Solution:

$$x^2 = 16y$$

$$(12)^2 = 16y$$

$$y = 9$$

$$x = \frac{3}{4}(12)$$

$$\bar{x} = 9$$

---

84. Locate the centroid of the are bounded by the parabola $y^2=4x$, the line y=4 and the y-axis

Ans. 6/5, 3

Solution:

$$\bar{x} = \frac{3}{10}(4)$$

$$\bar{x} = \frac{6}{5}$$

$$\bar{y} = \frac{3}{4}(4)$$

$$\bar{y} = 3$$

---

85. Find the centroid of the area bounded by the curve $x^2=-(y-4)$, the x-axis and the y-axis on the first quadrant.

Ans. 3/4, 8/5

Solution:

$$(x-h)^2 = -4a(y-k)$$

$$h = 0 \qquad k = 4$$

when $y = 0$

$$X^2 = -(0.-4)$$

$$x^2 = 4$$

$$x = 2$$

$$\bar{x} = \left(\frac{3}{8}\right)(2) = \frac{3}{4}$$

$$\bar{y} = \frac{2}{5}(4) = \frac{8}{5}$$

86.

87. Given the area in the first quadrant bounded by $x^2=8y$, the line x=4 and the x-axis. What is the volume generated by revolving this area about the y-axis?

Ans. 50.265 cu. units

Solution:

$A = \frac{ab}{3}$

Using 2nd Prop. of Pappus

$A = \frac{4(2)}{3} = \frac{8}{3}$

$\bar{x} = 3/4(4) = 3$

$V = 2\pi \bar{x} A$

$V = \frac{2\pi(3)(8)}{3}$

$V = 50.265$ cu. units

Check by integration:

$V = \int_0^4 2\pi xy dx$

$V = \frac{2\pi}{8}\int_0^4 x x^2 dx$

$V = \frac{\pi}{4}\left[\frac{x^4}{4}\right]_0^4$

$V = 50.265$

---

87. Find the volume of common to the cylinders $x^2+y^2=9$ and $y^2+z^2=9$.

Ans. 144 cu. m.
Solution:

Use Prismoidal Formula

$A_m = 6(6) = 36$

$V = \frac{L}{6}(A_1 + 4A_m + A_2)$

$V = \frac{6}{6}[0 + 4(36) + 0]$

$V = 144$ cu. m.

---

88. The area on the first and second quadrant of the circle $x^2+y^2=36$ is revolved about the line y=6. What is the volume generated?

Ans. 1225.80 cu. units
Solution:

Using second proposition of Pappus

$V = A 2\pi r$

$V = \frac{\pi(6)^2}{2} 2\pi(3.45)$

$V = 1225.80$ cu. units

---

89. Given the area in the first quadrant bounded by $x^2=8y$, the line y-2=0 and the y-axis. What is the volume generated when this area is revolved about the x-axis?

Ans. 40.21 cu. units
Solution:

$A = \frac{4(2)(2)}{3} = \frac{16}{3}$

$V = 2\pi \bar{y} A \qquad x = \sqrt{8}$

$V = 2\pi \frac{(6)}{5}\frac{(16)}{3} \quad x = 2\sqrt{2}\, y^{1/2}$

$V = 40.21$ cu. units

Check by integration:

$V = \int_0^2 2\pi y x dx$

$V = 2\pi \int_0^2 2\sqrt{2}\, y\, y^{1/2} dy$

$V = 4\pi\sqrt{2}\int_0^2 y^{3/2} dy$

$V = 4\pi\sqrt{2}\left[\frac{2}{5} y^{5/2}\right]_0^2$

$V = 40.21$ cu. units

---

90. Given is the area in the first quadrant bounded by $x^2=8y$, the line y-2=0 and the y-axis. What is the volume generated when this area is revolved about the line y-2=0.

Ans. 26.81

Solution:

$X^2=8y$

$X^2=8(2)$

$X = \pm 4$

Using Second Proposition of Pappus.

$V = A\, 2\pi y$

$V = \frac{2}{3}(2)(4)\, 2\pi\left(\frac{4}{5}\right)$

$V = 26.81$

---

91. Evaluate the integral of $e^{x^3} x^3 x^2 dx$ from 0 to 2.

Ans. 993.32
Solution:

$\int_0^2 e^{x^3} x^2 dx = \frac{1}{3}\int_0^2 e^{x^3} 3x^2 dx = \frac{1}{3}\left[e^{x^3}\right]_0^2$

$= \frac{1}{3}\left[e^8 - e^0\right]$

$= 993.32$

---

92. Determine the moment of inertia with respect to x-axis of the region in the first quadrant which is bounded by the curve $y^2=4x$, the line y=2 and the y-axis.

Ans. 1.6
Solution:

$I_x = Ay^2$

$I_x = \int_0^2 x\,dy\,y^2$

$I_x = \int_0^2 x\,y^2\,dy$

$I_x = \int_0^2 \frac{y^2}{4} y^2 d$

$I_x = \left[\frac{y^5}{4(5)}\right]_0^2$

$I_x = 1.6$

93. Find the moment of inertia, with respect to x-axis of the area bounded by the parabola y^2=4x and the line x=1.

Ans. 2.13
Solution:

$I_x = s\int_0^2 y^2(1-x)\,dy$

$I_x = 2\int_0^2 y^2\left(1-\frac{y^2}{4}\right)dy$

$I_x = 2\int_0^2 \left(y^2-\frac{y^4}{4}\right)dy$

$I_x = 2\left[\frac{y^3}{3}-\frac{y^5}{20}\right]_0^2$

$I_x = 2.13$

94. Find y=f(x) if dy/dx=8.60$x^{1.15}$

Ans. 4$x^{2.15}$ + C

Solution:

$dy = 8.60\,x^{1.15}\,dx$

$y = \int 8.60\,x^{1.15}\,dx$

$y = \frac{8.60\,x^{2.15}}{2.15}+C$

$y = 4x^{2.15}+C$

95. Evaluate:

$\int_0^8 xy\,dx$ subject to the functional relation $x = t^3$ and $y = t^2$.

Ans. 96

Solution:

$x = t^3$

$dx = 3t^2dt$

when $x = 0$

$0 = t^3$

$t = 0$

when $x = 8$

$8 = t^3$

$t = 2$

$\int_0^8 xy\,dx = \int_0^2 t^3\,t^2\,3t^2\,dt$

$3\int t^7\,dt = 3\left.\frac{t^8}{8}\right]_0^2$

$= \frac{3(2)^8}{8}$

$= 96$

96. Evaluate the integral of Cos (ln x) dx/x

Ans. Sin ln x + C
Solution:

$\int \text{Cos}\ln x\frac{dx}{x}$

$\int \text{Cos}\,u\,du = \text{Sin}\,u + C$

$u = \ln x$

$du = \frac{dx}{x}$

$\int \text{Cos}(\ln x)\frac{dx}{x} = \text{Sin}\ln x + C$

97. A body moves along a straight path such that its velocity is given by the expression v = 2 +1/2t + 1/3 t^2 where v is in m/s and t is in sec. If the distance traveled after 1 sec. is 2.361 m., find the distance it travels at the end of 3 sec.

Ans. 11.25 m.
Solution:

$$v = \frac{dS}{dt}$$
$$dS = v\,dt$$
$$\int dS = \int\left(2 + \frac{1}{2}t + \frac{1}{3}t^2\right)dt$$
$$S = 2t + \frac{1}{4}t^2 + \frac{1}{9}t^3 + C$$
$$\text{when } t = 1,\ S = 2.361$$
$$\text{thus, } 2.361 = 2(1) + \frac{1}{4}(1)^2 + \frac{1}{9}(1)^3 + C$$
$$C = 0$$
$$\therefore S = 2t + \frac{1}{4}t^2 + \frac{1}{9}t^3$$
$$\text{when } t = 3$$
$$S = 2(3) + \frac{1}{4}(3)^2 + \frac{1}{9}(3)^3$$
$$S = 11.25\,m.$$

98. Evaluate the following integral

$$\int_0^2 3^{2x}dx$$

Ans. 36.41
Solution:

$$\int_0^2 3^{2x}\,dx = \frac{1}{2}\int_0^2 3^{2x}\,2dx$$
$$= \frac{1}{2}\left[\frac{3^{2x}}{\ln 3}\right]_0^2 = \frac{1}{2\ln 3}\left[3^4 - 3^\circ\right]$$
$$= \frac{1}{2\ln 3}(81 - 1)$$
$$= 36.41$$

99. Evaluate $\int_0^1 \frac{3^x}{e^x}dx$

Ans. 1.051

$$\int_0^1 \frac{3^x}{e^x}dx = \int_0^1 \left(\frac{3}{e}\right)^x dx$$
$$\int_0^1 (1.1036)\,xdx$$
$$= \int a^u\,du = \frac{a^u}{\ln a}$$
$$\int_0^1 (1.1036)^x dx = \left.\frac{1.1036^x}{\ln 1.1036}\right]_0^1 = 1.051$$

100. A 5n lb. monkey is attached to a 20 ft. hanging rope that weighs 0.3 lb/ft. The monkey climbs the rope up to the top. How much work has it done.

Ans. 160 ft.-lb.

Solution:

$$W = \int_0^{20} (5 + 0.3x)\,dx$$
$$W = \left[5x + \frac{0.3x^2}{2}\right]_0^{20}$$
$$W = \left[5(20) + \frac{0.3(20)^2}{2}\right]$$
$$W = 160\,ft. - lb.$$

101. A 40 kg. block is resting on an inliced plane making an angle of θ from the horizontal. Coefficient of friction is 0.60, find the value of θ when a force P=36.23 is applied to cause motion upward along the plane.

Ans. 20°
Solution:

$$F = \mu N$$
$$F = 0.60(40)\,Cos\theta$$
$$F = 24\,Cos\theta$$
$$P = \omega Sin\theta + F$$
$$36.23 = 40Sin\theta + 24Cos\theta$$
$$\text{Try } \theta = 20^\circ$$
$$36.23 = 36.233\ \text{ok}$$
$$\text{use } \theta = 20^\circ$$

102. A spring with a natural length of 10 cm, is stretched by 1/2 cm. by a 12 Newton force. Find the work done in stretching the spring from 10 cm. to 18 cm. Express your answer in joules.

Ans. 7.68 Joules
Solution:

Using Hookes Law:

$F = kx$

$12 = k(0.5)$

$k = 24$

$F = 24x$

$W = \int_0^8 F\,dx$

$W = \int_0^8 24x\,dx$

$F = \left.\frac{24x^2}{2}\right]_0^8$

$F = 12(8)^2$

$F = 768\ \text{N.cm.}$

$F = \frac{768}{100}$

$F = 7.68\ \text{N.m.}$

$F = 7.68\ \text{Joules}$

103.A rectangular block aving a width of 8 cm and a height of 20 cm. is resting on a horizontal plane. If the coefficient of friction between the horizontal plane and the block is 0.40, at what point above the horizontal plane should a horizontal force P will be applied at which tipping will occur?

Ans. 10 cm.
Solution:

$P = F$

$P = 0.40W$

$\sum M_A = 0$

$W(4) = P(h)$

$4W = 0.40Wh$

$h = 10\ \text{cm.}$

104.A block weighing 400 kg is placed on an inclined plane making an angle of θ from the horizontal. If the coefficient of friction between the block and the inclined plane is 0.30, find the value of θ, when the block impends to slide downward.

Ans. 16.70 °
Solution:

$W = \text{Cos}\,\theta = N$

$F = \mu N$

$F = W\,\text{Sin}\theta$

$\mu N = W\,\text{Sin}\theta$

$\mu W\,\text{Cos}\theta = W\,\text{Sin}\,\theta$

$\tan\theta = \mu$

$\tan\theta = 0.30$

$\theta = 16.70°$

---

105. A certain cable is suspended between two supports at the same elevation and 50 m. apart. The load is 50 N per meter horizontal length including the weight of the cable. The sag of the cable is 3 m. Calculate the total length of the cable.

Ans. 50.4758 m.
Solution:

$S = L + \frac{8d^2}{3L} - \frac{32d^4}{5L^3}$

$S = 50 + \frac{8(3)^2}{3(50)} - \frac{32(3)^4}{5(50)^3}$

$S = 50.4758\ \text{m.}$

---

106. The cable supported at 2 points of same level has a unit weight W of 0.02 kg. per meter of horizontal distance. The allowable sag is 0.02 m. and a maximum tension at the lowest point of 1200 kg. and a factor of safety of 2. Calculate the allowable spacing of the poles assuming a parabolic cable.

Ans. 69.28 m.
Solution:

$T = \frac{1200}{2}$

$T = 600$

$Td = W\left(\frac{L}{2}\right)\left(\frac{L}{4}\right)$

$(600)(0.02) = \frac{0.02L^2}{8}$

$L = 69.28\ \text{m.}$

107.A cable 800 m. long wiehing 1.5 kN/m has tension of 750 kN at each end. Compute the maximum sag of the cable.

Ans. 200 m.
Solution:

$T = \omega y$

$750 = 1.5y$

$y = 500$

$y^2 = s^2 + c^2$

$(500)^2 = (400)^2 + c^2$

$c = 300$

$d = y - c = 500 - 300$

$d = 200\ \text{m.}\ (\text{sag of cable})$

108.An object experiences rectilinear acceleration a(t)=10-2t . How far does it travel in 6 seconds if its initial velocity is 10m/s.
Ans. 168 m.
Solution:

$$a = \frac{dV}{dt}$$
$$\frac{dV}{dt} = 10 - 2t$$
$$dV = (10 - 2t)\,dt$$
$$\int_{10}^{v} dV = \int_0^t (10 - 2t)\,dt$$
$$V - 10 = 10t - \frac{2t^2}{2}$$
$$V - 10 = 10t - t^2$$
$$V = 10t - t^2 + 10$$
$$V = \frac{dS}{dt}$$
$$dS = V\,dt$$
$$\int_0^s dS = \int_0^6 \left(10t - t^2 + 10\right) dt$$
$$S = \left[\frac{10t^2}{2} - \frac{t^3}{3} + 10t\right]_0^6$$
$$S = 5(36) - \frac{(6)^3}{3} + 10(6)$$
$$S = 168\,\text{m.}$$

109. An object is accelerating to the right along a straight path at 2m/sec? The object begins with a velocity of 10 m/s to the left. How far does it travel in 15 seconds?

Ans. 125 m.

Solution:

$$V_2 = V_1 + at$$
$$0 = 10 - 2t$$
$$t = 5\,\text{sec.}$$
$$V_2^2 = V_1^2 + 2aS_1$$
$$0 = (10)^2 - 2(2)S_1$$
$$S_1 = 25\,\text{m.}$$
$$S_2 = V_2 t + \frac{1}{2}at^2$$
$$S_2 = 0 + \frac{1}{2}(2)(10)^2$$
$$S_2 = 100\,\text{m.}$$
$$\text{Total distance traveled} = 125\,\text{m.}$$

110.A shot is fired at an angle of 45 ° with the horizontal and a velocity of 300 fps. Calculate to the nearest, the rage of the projectile.
Ans. 932 yds.
Solution:

$$R = \frac{V^2 \sin 2\theta}{g}$$
$$R = \frac{(300)^2 \sin 90°}{2.2} = 2795\,\text{ft.}$$
$$R = \frac{2795}{3}$$
$$R = 932\,\text{yds.}$$

111.A solid disk flywheel (I=200 kg-m) is rorating with a speed of 900 rpm. What is the rotational kinetic energy?
Ans. 888 x 10^3 J
Solution:

$$KE = \frac{1}{2}I\omega^2$$
$$\omega = \frac{900(2\pi)}{60}$$
$$\omega = 94.25\,\text{rad/sec.}$$
$$KE = \frac{1}{2}(200)(94.25)^2$$
$$KE = 888264\,\text{J}$$

112.A bill for motorboat specifies the cost as P1,200 due at the end of 100 days but offers a 4% discount for cash in 30 fays. What is the highest rate, simple interest at which the buyer can afford to borrow money in order to take advantage of the discount?

Ans. 21.4%
Solution:

$$\text{Discount} = 0.04(1200)$$
$$\text{Discount} = P48$$
Amount to be paid on 30 days
$$= 1200 - 48 = P1152$$
$$\text{Diff. in no. of days} = 100 - 30$$
$$\text{Diff. in no. of days} = 70 \text{ days}$$
$$I = Prt$$
$$48 = 1152(r)\frac{(70)}{360}$$
$$r = 0.214$$
$$r = 21.4\%$$

113.A man borrowed P2000 from a bank and promise to pay the amount for one year. He recived only the amount of P1,920 after the bank collected an advance interest of P80. What was the rate of discount and the rate of interest that the bank collected in advance.
Ans. 4%, 4.17%
Solution:

$$\text{Rate of discount} = \frac{80}{2000}x100 = 4\%$$
$$\text{Rate of interest} = \frac{80}{1920}x100 = 4.17\%$$
Another solution:
$$I = \frac{d}{1-d} = \frac{0.04}{1-0.04}$$
$$I = 0.0417 = 4.17\%$$

114.Find the discount if P2000 is discounted for 6 months at 8% simple discount.
Ans. P 80.00
Solution:

$$\text{Discount} = \frac{2000(0.08)(6)}{12}$$
$$\text{Discount} = P80.00$$

115.If the rate of interest is 12% compounded annually, find the equivalent rate of interest if it is compounded quarterly.

Ans. 11.49%
Solution:

$$(1.12)^1 = \left(1 + \frac{i}{4}\right)^4$$
$$1 + \frac{i}{4} = 1.0287$$
$$i = 0.1149$$
$$i = 11.49\% \text{ (compounded quarterly)}$$

116.How long will it take money to double itself if invested at 5% compounded annually?

Ans. 14 years
Solution:

$$F = P(1+i)^n$$
$$2P = P(1.05)^n$$
$$2 = (1.05)^n$$
$$\log 2 = n \log 1.05$$
$$n = \frac{\log 2}{\log 1.05}$$
$$n = 14.2$$

117.A person borrows 10,000 from a credit union. In repaying the debt he has to pay 500 at the end of every 3 months on the principal and a simple interest of 18% on the amount outstanding at that time. Determine the total amount he has paid after paying all his debt.

Ans. 9,725.00

Solution:

The total amount paid on the principal in 1 year will be 4(500)=2,000. Thus, he will be able to repay the debt after 10,000/2,000 = 5 years. He will have made 20 payments. The interest rate for each period = 18%/4=4.5%=0.045.

Interest paid on first payment = 0.045(10,000)=450.00
Interest paid on 2nd payment=0.045(9,500)=427.50
Interest paid on 3rd payment = 0.045 (9,000)=405.00

Interest paid on 20th payment=0.045(500)=22.50
Total interest paid=450.00+427.50+… + 22.50
This is an arithmetic progression whose sum is 20/2(450.00+22.50) = 4,725.00

Total amount paid = 5,000 + 4, 725 = 9,725.00

118.A man borrows 500 for one year at a discount rate of 8% per annum. a.) How much does he actually receive? b.) What rate of interest is he actually paying?

Ans. a.) 460.00 b.) 8.70%

Solution:

$$a.)\ 500 - 0.08(500) = 460.00$$
$$b.)\ i = \frac{d}{1-d} = \frac{0.08}{1-0.08} = 0.08696 = 8.696\%$$

119.A man borrows 2,000 for 18 months at a discount rate of 12% per annum. a.) How much does he actually receive? b.) What rate of interest is he actually paying ?

Ans. a.) 1,640.00 b.) 13.64%
Solution:

$$a.)\ P = 2,000 - 0.12(1.5)\,2,000$$
$$P = 1,640.00$$
$$b.)\ i = \frac{d}{1-d} = \frac{0.12}{1-0.12}$$
$$i = 0.1364$$
$$i = 13.64\%$$

120. A man borrows some money at a discount rate of 15% per annum for one year. If he wishes to receive 1,000, how much must he borrow?

Ans. 1,176.47
Solution:

Let $P =$ amount to be borrowed
$0.15P =$ amount to be deducted from P.
$$P - 0.15P = 1,000$$
$$P = \frac{1,000}{0.85}$$
$$P = 1,176.47$$

---

121. Find the amount at the end of 8 years if 1,000 is invested at 6% per annum compounded a.) monthly b.) quarterly

Ans. a.) 1,614.14 b.) 1,610.32

Solution:

$$a.)\ P = 1,000 \qquad i = \frac{6\%}{12} = 0.005$$
$$n = 8(12) = 96$$
$$F = P(1+i)^n = 1,000(1+0.005)^{96}$$
$$F = 1,000(1.614143) = 1,614.14$$
$$b.)\ P = 1,000 \quad i = \frac{6\%}{4} = 1.5\% = 0.015$$
$$n = 8(4) = 32$$
$$F = P(1+i)^n = 1,000(1+0.015)^{32}$$
$$= 1,000(1.61032) = 1,610.32$$

---

122. How much must be invested now in order to have 10,000 at the end of 10 years if interest is paid at the rate of 6% per annum compounded a.) annually b.) semi-annually?
Ans. a.) 5,583.95 b.) 5,536.76

$$a.)\ F = 10,000 \quad i = 6\% = 0.06 \quad n = 10 \quad P = ?$$
$$F = P(1+i)^n$$
$$10,000 = P(1+0.06)^{10} = (1.06)^{10}P$$
$$P = \frac{10,000}{(1.06)^{10}} = 10,000(1.06)-10$$
$$= 10,000(0.5583947)$$
$$= 5,583.95$$
$$b.)\ F = 10,000 \quad i = \frac{6\%}{2} = 3\% = 0.03 \quad n = 10(2) = 20$$
$$F = P(1+i)^n$$
$$10,000 = P(1+0.03)^{20}$$
$$P = 10,000(1.03)^{-20}$$
$$P = 10,000(0.5536757)$$
$$P = 5,536.76$$

---

124 How many years will it take for an investment a.) to double itself, b.) to triple itself if invested at the rate of 4% compounded annually?

Ans. a.) 18 years b.) 28 years

Solution:

$$a.)\ \text{Consider } P = 1.00 \quad F = 2.00 \quad i = 4\% = 0.04 \quad n = ?$$
$$F = P(1+i)^n$$
$$2.00 = 1.00(1+0.04)^n$$
$$(1.04)^n = 2 \quad n\log 1.04 = \log_2$$
$$n = 17.67 \text{ say } 18 \text{ years}$$
$$b.)\ \text{Let } P = 1.00 \quad F = 3.00 \quad i = 0.04 \quad n = ?$$
$$F = P(1+i)^n$$
$$3.00 = 1.00(1.04)^n$$
$$(1.04)^n = 3 \quad n\log 1.04 = \log 3$$
$$n = 28.01 \text{ say } 28 \text{ years}$$

---

125. Find the effective rate if interest is compounded monthly at the nominal rate of 12% per annum.

Ans. 12.68%

Solution:

$$i = \frac{12\%}{12} = 1\% = 0.01 \quad n = 12\,\text{months / year}$$

$$\text{Effective rate} = (1+i)^n - 1 = (1.01)^{12} - 1$$
$$= 0.1268 = 12.68\%$$

126. Find the effective rate if interest is compounded quarterly at the nominal rate of 9.6% per annum.

Ans. 9.95%

Solution:

$$i = \frac{9.6\%}{4} = 2.4\% = 0.024 \quad n = 4\,\text{quarters / year}$$

$$\text{Effective rate} = (1+i)^n - 1 = (1.024)^4 - 1$$

$$\text{Effective rate} = 0.09951 = 9.95\%$$

127. A compressor can be purchased with a down payment of 8,000 and equal installments of 600 each paid at the end of every month for the next 12 months. If money is worth 12% compounded monthly, determine the equivalent cash price of the compressor.

Ans. 14,753.05

Solution:

The equivalent cash price of the compressor is the present value o

Hence,

P=equivalent cash price=8,000+600(P/A,1%,12)

$$P = 8{,}000 + 600\,\frac{1-(1.01)-12}{0.01} = 8{,}000 + 6{,}753.05$$

$$P = 14{,}753.05$$

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