

MATHEMATICS (MCQs)

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Mathematics (MCQs)

Numbers

Numbers: In decimal number system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called digits, to represent any number.

Note: A group of figures, denoting a number is called numeral.

Types of Numbers

Natural Numbers: Numbers which we use for counting the objects are known as Natural numbers. It is denoted by ' N '.

$$N = \{1, 2, 3, 4, \dots\}$$

Whole Numbers: All Natural numbers together with zero form the set of all whole numbers. It is denoted by ' W '.

$$W = \{0, 1, 2, 3, \dots\}$$

Integers: The set of numbers which consists of whole numbers and negative numbers is known as integers. It is denoted by Z .

$$Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$$

Positive Integers: The set $Z^+ = \{1, 2, 3, 4, \dots\}$ is the set of all positive integers. It is clear that positive integers and Natural numbers are synonyms.

Negative Integers: The set $Z^- = \{-1, -2, -3, \dots\}$ is the set of all negative integers.

Non-Negative Integers: The set $\{0, 1, 2, 3, \dots\}$ is a set of non-negative integers.

Non-Positive Integers: The set $\{0, -1, -2, -3, \dots\}$ is the set of non-positive integers.

Even Numbers: The numbers which are divisible by 2 are called Even numbers.

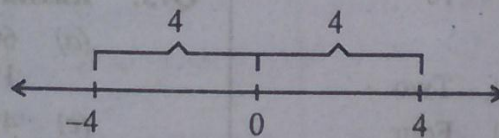
$$E = \{2, 4, 6, \dots\}$$

Odd Numbers: The numbers which are not divisible by 2 are called Odd numbers.

$$O = \{3, 9, 11, 17, 19, \dots\}$$

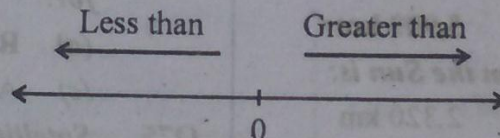
ABSOLUTE VALUES: The absolute value of a number or expression is always positive because it is the difference a number is may from zero on a number line.

Example:



$$|4| = |-4| = 4 \text{ units away from } 0$$

Number Lines and Signed Numbers: The concept of number line is very simple. Less than is to the left and greater than is to the right.



Sometimes, we confused about the values of negative numbers. To keep things simple, remember, if

$$a > b$$

$$\text{then } -b > -a$$

Example: If $5 > 3$ then $-3 > -5$

Multiple Choice Questions (MCQs)

- Q1.** How many numbers between 200 and 500 are divisible by 13?
 (A) 23 (B) 17
 (C) 15 (D) 32
- Q2.** The first five multiples of 17 are:
 (A) 0, 1, 17, 34, 51 (B) 17, 34, 51, 68, 85
 (C) 38, 57, 76, 95, 114 (D) None of these
- Q3.** The number which is divisible by 7 but not by 14 is:
 (A) 21 (B) 12
 (C) 71 (D) None of these
- Q4.** The total number of even prime numbers is:
 (A) 0 (B) 1
 (C) 2 (D) None of these
- Q5.** The least prime number is:
 (A) 0 (B) 1
 (C) 2 (D) 3
- Q6.** The smallest member of set W is:
 (A) 0 (B) 1
 (C) 2 (D) 3
- Q7.** The smallest even number of three digits is:
 (A) 98 (B) 102
 (C) 998 (D) 100
- Q8.** The smallest 4-digit number using 7, 0, 8 and 9 is:
 (A) 0879 (B) 0789
 (C) 0978 (D) 7890
- Q9.** The cube of $\frac{1}{2}$ is:
 (A) $\frac{1}{4}$ (B) $\frac{1}{8}$
 (C) $\frac{1}{2}$ (D) $\frac{1}{16}$
- Q10.** $3 - 7 =$
 (A) -7 (B) 7
 (C) -4 (D) 4
- Q11.** If $5x + 16 = 0$, then $15 | x |$ equals one of the following:
 (A) $16x$ (B) $-16x$
 (C) 16 (D) $15x$
- Q12.** Which one of the following equals the product of exactly two prime numbers?
 (A) 11.6 (B) 14.23
 (C) 17.21 (D) 13.23
- Q13.** A number whose fifth part increased by 5 is equal to its fourth part diminished by 5, is:
 (A) 160 (B) 180
 (C) 200 (D) 220
- Q14.** If $(5^a)(5^b) = \frac{5^c}{5^d}$, what is d in terms of a, b and c ?
 (A) $a + b - c$ (B) $a - b + c$
 (C) $a + b + c$ (D) $c - a - b$
- Q15.** Which of the following is equal to $(3^8 \times 3^3)^{10}$?
 (A) 3^{720} (B) 3^{170}
 (C) 3^{27} (D) 3^{98}
- Q16.** If $0 < p < 1$, which of the following lists the numbers are in increasing order?
 (A) p, \sqrt{p}, p^2 (B) p, p^2, \sqrt{p}
 (C) \sqrt{p}, p, p^2 (D) p^2, p, \sqrt{p}
- Q17.** The value of x satisfying $\sqrt{5 + \sqrt[3]{x}} = 3$ is:
 (A) 64 (B) 27
 (C) 125 (D) 9
- Q18.** If $x^{\sqrt{x}} = (x\sqrt{x})^x$, then $x =$
 (A) $\frac{1}{2}$ (B) $\frac{9}{4}$
 (C) $\frac{3}{2}$ (D) $\frac{1}{4}$
- Q19.** $(16)^{7/4}$ is equal to:
 (A) 28 (B) 128
 (C) 27 (D) None of these
- Q20.** $\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is:
 (A) 30 (B) 60
 (C) 75 (D) 90

Explanatory Answers

Q1.(A) Number of numbers up to 200 which are divisible by 13
 $= \frac{200}{13} = 15 + \frac{5}{13}$, i.e., 15

Number of numbers up to 500 which are divisible by 13
 $= \frac{500}{13} = 38 + \frac{6}{13}$ i.e., 38

The required numbers = $38 - 15 = 23$
Hence, the correct answer is choice A.

Q2.(B) The first five multiples of 17 are

$$\begin{aligned} 17 \times 1 &= 17 \\ 17 \times 2 &= 34 \\ 17 \times 3 &= 51 \\ 17 \times 4 &= 68 \\ 17 \times 5 &= 85 \end{aligned}$$

First five multiples of 17 are 17, 34, 51, 68 and 85.

Q3.(A) The number which is divisible by 7 but not by 14 is 21. Hence, the correct answer is choice A.

Q4.(B) There is only one even prime number, namely 2. Hence, the correct answer is choice C.

Q5.(C) The least prime number is 2. Hence, the correct answer is choice C.

Q6.(A) 0 is the smallest member of the set W . Hence, the correct choice is A.

Q7.(D) The smallest even number of three digits is 100. The correct choice is choice D.

Q8.(B) Using 0, 7, 8, 9, the smallest number is 0789. Hence, the correct answer is choice B.

Q9.(B)
$$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1 \times 1 \times 1}{2 \times 2 \times 2} = \frac{1}{8}$$

Correct answer is choice B.

Q10.(C) $3 + (-7) = 3 - 7 = -4$

Correct answer is choice C.

Q11.(C) Solving the given equation

$$\begin{aligned} 15x + 16 &= 0 \\ \Rightarrow 15x &= -16 \\ \Rightarrow x &= \frac{-16}{15} \end{aligned}$$

Substituting this value in $15|x|$ yields

$$\begin{aligned} 15|x| &= \\ 15\left|\frac{-16}{15}\right| &= 15 \times \frac{16}{15} \\ &= 16 \end{aligned}$$

Correct answer is choice C.

Q12.(D) Since 13 . 23 cannot be further factored and is itself the product of two primes. Hence, correct answer is choice D.

Q13.(C)
$$\frac{x}{5} + 5 = \frac{x}{4} - 5 \Rightarrow \frac{x}{4} - \frac{x}{5} = 10$$

$$\Rightarrow 5x - 4x = 200$$

$$\Rightarrow x = 200$$

Hence, the correct answer is choice C.

Q14.(D)
$$(5^a)(5^b) = \frac{5^c}{5^d}$$

$$5^{a+b} = 5^{c-d} \quad (\text{By power rule})$$

$$\Rightarrow a + b = c - d$$

$$\Rightarrow d = c - a - b$$

Hence, the correct answer is choice D.

Q15.(B) Given that, $(3^8 \times 3^9)^{10}$
$$= (3^{8+9})^{10} \quad (\text{By power rule})$$

$$= (3^{17})^{10}$$

$$= 3^{17 \times 10}$$

$$= 3^{170}$$

Hence, the correct answer is choice B.

Q16.(D) For any number p , between 0 and 1
$$p^2 < p \text{ and } p < \sqrt{p}$$

Hence, the correct answer is choice D.

Q17.(A)
$$\sqrt{5 + 3\sqrt{x}} = 3$$

$$5 + 3 = 9 \quad (\text{Squaring both sides})$$

$$3\sqrt{x} = 9 - 5$$

$$3\sqrt{x} = 4$$

$$((x)^{1/3})^3 = (4)^3$$

$$x^{1/3 \times 3} = 4 \times 4 \times 4$$

$$x = 64$$

Hence, the correct answer is choice A.

Q18.(B)
$$x^x \sqrt{x} = (x\sqrt{x})^x$$

$$x^x \sqrt{x} = (x \cdot x^{1/2})^x$$

$$\Rightarrow x^x \sqrt{x} = (x^{3/2})^x$$

$$\Rightarrow x^x \sqrt{x} = (x^{3x/2})$$

$$\Rightarrow x\sqrt{x} = \frac{3x}{2}$$

$$\Rightarrow \sqrt{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{9}{4}$$

Hence, the correct answer is choice B.

Q19.(B)
$$(16)^{7/4}$$

$$= (2^4)^{7/4}$$

$$= 2^{4 \times 7/4}$$

$$= 2^7$$

$$= 128$$

Hence, the correct answer is choice B.

Q20.(B)
$$\frac{4}{5}x - \frac{2}{3}x = 8$$

$$\Rightarrow 12x - 10x = 120$$

$$\Rightarrow 2x = 120$$

$$\Rightarrow x = 60$$

Hence, the correct answer is choice B.

Multiplication & Division

MULTIPLICATION

Multiplication is a short method of adding the same number repeatedly.

Some Important Results:

(i) Multiplication of a given number by

yields $10^n - 1$

9, 99, 999, etc.

Model Examples:

Example 1:

Multiply

- (a) 2435 by 99 (b) 3791 by 999 (c) 7847 by 9999
 (d) 6788 by 99999

Solution:

- (a) $2435 \times 99 = 243500 - 2435 = 241065$
 (b) $3791 \times 999 = 3791000 - 3791 = 3787209$
 (c) $7847 \times 9999 = 78470000 - 7847 = 78462153$
 (d) $6788 \times 99999 = 678800000 - 6788 = 678793212$

(ii) Multiplication of a given numbers by

yields $10^n + 1$

11, 101, 1001, 10001, etc.

Example 2:

Multiply

- (a) 2123 by 11 (b) 7887 by 101
 (c) 9121 by 1001 (d) 6153 by 10001

Solution:

- (a) $2123 \times 11 = 21230 + 2123 = 23353$
 (b) $7887 \times 101 = 788700 + 7887 = 796587$
 (c) $9121 \times 1001 = 9121000 + 9121 = 9130121$
 (d) $6153 \times 10001 = 61530000 + 6153 = 61536153$

DIVISION

The process of subtraction of the same number form a given number for a few times is called division (\div), i.e.,

$$6 \div 2 = 3$$

(2 can be subtracted 3 times from 6)

Model Examples:

Q1. Multiply 63987 by 91763 is not more than 3 lines.

Solution:

$$\begin{array}{r}
 63986 \\
 (\times) \underline{91763} \\
 4031118 \quad \text{Multiplication by } 63 \\
 447902 \times \times \quad \text{Multiplication by } 700 \\
 5822726 \times \times \times \quad \text{Multiplication by } 91000 \\
 \hline
 5871547318
 \end{array}$$

Q2. Find the number, one-sixth of which exceeds its one-ninth by 654.

Solution: Let the number be x

$$\begin{aligned} \therefore \frac{x}{6} - \frac{x}{9} &= 654 \\ \Rightarrow \frac{x}{18} &= 654 \\ \Rightarrow x &= 654 \times 18 = 11772 \end{aligned}$$

Ans. *****

Highest Common Factor & Least Common Multiple

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

Methods of finding H.C.F.

(i) By Prime Factors.

Resolve the given number into their prime factors. The product of all prime common factors is known as H.C.F.

Model Example

Find the H.C.F. of 630, 1050 and 1260.

Solution: $630 = 2.3.3.5.7$

$$1050 = 2.3.5.5.7$$

$$1260 = 2.2.3.3.5.7$$

\therefore H.C.F. is $2.3.5.7 = 210$. **Ans.**

(ii) By Division:

Find the H.C.F. of 5133 and 3953

$$\begin{array}{r} 3953 \overline{)5133} \quad (1 \\ \underline{3953} \\ 1180 \\ \underline{1180} \\ 3540 \\ \underline{3540} \\ 413 \\ \underline{413} \\ 826 \\ \underline{826} \\ 354 \\ \underline{354} \\ 59 \\ \underline{59} \\ 354 \\ \underline{354} \\ \\ \\ \\ \\ \end{array}$$

Various Steps:

- Step I.** Dividing the greatest number by the lesser, we get the remainder 1180.
- Step II.** Dividing the previous divisor 3953 by 1180, we get the remainder 413.
- Step III.** Dividing the previous divisor 1180 by 413, we get the remainder 354.
- Step IV.** Dividing the previous divisor 413 by 354, we get the remainder 59.
- Step V.** Dividing the previous divisor 354 by 59, we get no remainder.

\therefore The last divisor 59 is the H.C.F.

H.C.F. is also known as Greatest Common Measure (G.C.M.).

LEAST COMMON MULTIPLE (L.C.M)

The Least Common Multiple of two or more given numbers is the least number, which is exactly divisible by

each of them.

Methods of Finding L.C.M.

(i) **By Factors.** Resolve the given numbers into prime factors, and find the product of the highest powers of all the factors that occur in the given number. The product will be the required L.C.M.

Model Example

Q1. Find the L.C.M. of 70, 80, 90.

Solution: $70 = 2 \times 5 \times 7$

$80 = 2^4 \times 5$

$90 = 2 \times 3^2 \times 5$

L.C.M. = $2^4 \cdot 3^2 \cdot 5 \cdot 7 = 5040$ Ans.

(ii) **With the help of H.C.F.** The product of two numbers is equal to the product of their L.C.M. and H.C.F.

$$\begin{aligned} \therefore \text{L.C.M. of two numbers} \\ &= \frac{\text{Product of numbers}}{\text{H.C.F.}} \end{aligned}$$

L.C.M. and H.C.F. of Fractions.

Rule: First express the fractions in their lowest terms, then

L.C.M. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F. of two or more fractions

Rule: First express the given fractions in their lowest terms, then

$$= \frac{\text{H.C.F. of numerators}}{\text{L.C.M. of denominators}}$$

Model Examples

Example 1: The H.C.F. of two numbers is 34 and their L.C.M. is 4284. If one of the numbers is 204, find the other.

Solution: As product of 2 numbers = their H.C.F. \times L.C.M.

The other number is $= \frac{34 \times 4284}{204}$

$= 714$ Ans.

Example 2: What is the highest number of four digits which will leave a remainder of 1 when divided by any of numbers 6, 9, 12, 15, or 18?

Solution: L.C.M. of 6, 9, 12, 15, 18 = 180

Greatest no. of 4 digits = 9999

Greatest no. of 4 digits divisible by

$180 = 9999 - 99 = 9900$

$$\begin{array}{r} 55 \\ 180 \overline{) 9999} \\ \underline{900} \\ 999 \\ \underline{900} \\ 99 \end{array}$$

\therefore Req'd. No. = $9900 + 1 = 9901$ Ans.

Multiple Choice Questions (MCQs)

- Q1.** A neon sign flashes every 3 seconds, another sign flashes every 5 seconds, and a third flashes every 7 seconds. If they all flash together, how many seconds will pass before they all flash simultaneously again?
- (A) 15 seconds (B) 35 seconds
(C) 105 seconds (D) 21 seconds
- Q2.** The greatest number which exactly divides 1155 and 735 is:
- (A) 25 (B) 5
(C) 15 (D) 105
- Q3.** The least number which when divided by 35, leaves remainder of 25; when divided by 45 leaves a remainder of 35 and when divided by 55 leaves 45 as remainder, is:
- (A) 3455 (B) 3465
(C) 3475 (D) 10
- Q4.** The L.C.M of 12, 20, 24, 32 is:
- (A) 240 (B) 360
(C) 480 (D) 600
- Q5.** How many whole bricks $6 \times 12 \times 24$ cm will be sufficient to construct a solid cube of minimum size?
- (A) 4 (B) 6
(C) 8 (D) 12
- Q6.** If the L.C.M and H.C.F of two numbers are 150 and 30 respectively, and one of the numbers is 18, find the other number?
- (A) 250 (B) 180
(C) 150 (D) 170
- Q7.** The product of two numbers is 2500. If their L.C.M is 125, then their H.C.F is:
- (A) 20 (B) 250
(C) 125 (D) None of these
- Q8.** It takes Riaz 30 minutes to mark a paper. Razi only need 25 minutes to mark a paper. If they both start marking papers at 11 : 00 AM, what is the first time they will finish marking a paper at the same time?
- (A) 12 : 30 (B) 12 : 45
(C) 1 : 30 (D) 12 : 25
- Q9.** Sonia buys two off-cuts of ribbon in a sale. One is 153 cm long. The other is 204 cm long. She cuts them so that she ends up with a number of pieces all the same length. What is the greatest length each piece can be?
- (A) 39 cm (B) 6 cm
(C) 17 cm (D) 51 cm
- Q10.** A farmer wants to fence a triangular field. He plans to put a fencing post in each corner and place other posts at equal distance along its sides. He wants the posts to be as far apart as possible. The sides of the field are 477 feet 2412 feet and 639 feet long. How far apart will the posts be?
- (A) 18 feet (B) 9 feet
(C) 27 feet (D) 159 feet
- Q11.** Find the greatest number of 4 digits which when divided by 18, 24, 30 and 36 leaves a remainder 17 in each case.
- (A) 360 (B) 9360
(C) 3600 (D) 9737
- Q12.** The least number which when divided by 12, 15 and 18 leaves 5 as remainder in each case is:
- (A) 180 (B) 175
(C) 185 (D) 125
- Q13.** The greatest number which divides 2400 and 3600 leaving 48 and 60 respectively, as remainder is:
- (A) 9 (B) 7
(C) 17 (D) 12
- Q14.** Ahmed has a rectangular garden measuring 4.32 m by 3.36 m. He wants to divide it into square plots of equal size. What is largest sized square he can use?
- (A) 0.24 (B) $\sqrt{3}$
(C) 0.48 m (D) 0.16
- Q15.** The chairs in the school hall can be set out in 25 equal rows or in 40 equal rows or in 120 equal rows are:
- (A) 600 (B) 400
(C) 40 (D) 80
- Q16.** Three bells toll after intervals of 6, 9 and 15 minutes, respectively. If they toll together at 5 p.m., when will they toll together next?
- (A) 6 : 30 (B) 5 : 30
(C) 6 : 45 (D) 5 : 45

Explanatory Answers

Q1. (C) The L.C.M of 3, 5 and 7 will give the answer

$$\begin{array}{r|l} 3 & 3-5-7 \\ \hline 5 & 1-5-7 \\ \hline 7 & 1-1-7 \\ \hline & 1-1-1 \end{array}$$

= $3 \times 5 \times 7 = 105$

Q2. (D) The required number is the H.C.F of 1155 and 735

$$\begin{array}{r} 735 \overline{)1155} \\ \underline{-735} \quad 1 \\ 420 \overline{)735} \\ \underline{-420} \quad 1 \\ 315 \overline{)420} \\ \underline{-315} \quad 3 \\ 105 \overline{)315} \\ \underline{-315} \\ \times \end{array}$$

The greatest number required is

105.

Q3. (A) The least number which is completely divided by 35, 45 and 55, is their L.C.M. which is 3465. We want to find the least

number which on dividing by 35, 45 and 55 leave remainders 25, 35 and 45 respectively i.e., 10 less than the quotient in each case. Hence such a number is $3465 - 10 = 3455$.

Q4. (C)

$$\begin{array}{r|l} 2 & 12-20-24-32 \\ \hline 2 & 6-10-12-16 \\ \hline 2 & 3-5-6-8 \\ \hline 2 & 3-5-3-4 \\ \hline 2 & 3-5-3-2 \\ \hline 3 & 3-5-3-1 \\ \hline 5 & 1-5-1-1 \\ \hline & 1-1-1-1 \end{array}$$

The L.C.M. of 12, 20, 24 and 32 is

$2^5 \times 3 \times 5 = 32 \times 3 \times 5 = 480$

Q5. (C) One edge of the minimum cube must be 24 cms, the least common multiple of 6, 12 and 24. Thus, it will have a volume of $24 \times 24 \times 24$ cubic centimeters which is equal to 8 bricks

i.e., $\frac{24 \times 24 \times 24}{6 \times 12 \times 24} = 8$

Q6. (A) Product of two numbers = L.C.M \times H.C.F

$18 \times 2\text{nd number} = 150 \times 30$

2nd number = $\frac{150 \times 30}{18}$

= 250

Q7. (A) Product of two numbers = L.C.M \times H.C.F

$2500 = 125 \times \text{H.C.F}$

$\Rightarrow \text{H.C.F} = \frac{2500}{125}$

= 20

Q8. (C) The question asks for the first time they will finish at the same time. So, we must find least common multiple

5	25 - 30
5	5 - 6
6	1 - 6
	1 - 1

$6 \times 5 \times 5 = 150$ minutes
 $= 2 : 30$ hours

So they will finish marking at 1 : 30 PM.

Q9. (D) The HCF of 153 and 204 gives the wanted length

3	153
3	51
17	17
	1

2	204
2	102
3	51
17	17
	1

$153 = 3 \times 3 \times 17$

$204 = 2 \times 2 \times 3 \times 17$

HCF = $17 \times 3 = 51$

Greatest length = 51cm

Q10. (B) The HCF of 477, 2412 and 639 gives the wanted length.

3	477
3	159
	53

2	2412
2	1206
3	603
3	201
	67

3	639
3	213
	71

$477 = 3 \times 3 \times 53$

$2412 = 2 \times 2 \times 3 \times 3 \times 67$

$639 = 3 \times 3 \times 71$

H.C.F = $3 \times 3 = 9$

Hence, the distance apart the posts is 9 feet.

Q11. (D) The number which is divisible by 18, 24, 30 and 36 is divisible by their L.C.M

2	18 - 24 - 30 - 36
2	9 - 12 - 15 - 18
3	9 - 6 - 15 - 9
3	3 - 2 - 5 - 3
2	1 - 2 - 5 - 1
5	1 - 1 - 5 - 1
	1 - 1 - 1 - 1

\therefore L.C.M = $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

The greatest number of 4-digits is 9999. Now we find the greatest multiple of 360 less than 9999.

$$\begin{array}{r} 27 \\ 360 \overline{) 9999} \\ \underline{720} \\ 2799 \\ \underline{2520} \\ 279 \end{array}$$

Thus $9999 - 279 = 9720$ is exactly divisible by 360. But the required number leaves a remainder 17 in each case. Hence, the number is

$$9720 + 17 = 9737$$

Q12. (C) Required number = L.C.M of 12, 15, 18

2	12, 15, 18
3	6 - 15 - 9
2	2 - 5 - 3
3	1 - 5 - 3
5	1 - 5 - 1
	1 - 1 - 1

$$\therefore \text{L.C.M} = 2 \times 2 \times 3 \times 3 \times 5 = 180$$

The required least number = $180 + 5 = 185$

Q13. (D) As 48 and 60 remainders when 2400 and 3600 are divided by the numbers $2400 - 48 = 2352$ and $3600 - 60 = 3540$ must be exactly divisible by the number.

The H.C.F of 2352 and 3540 is the required number.

2352	1	3540
2352	1	
1188		2352
1188	1	
1164		1188
1164	48	
24		1164
96		
204		
192	2	
12		24
24		
x		

- a. The H.C.F of 2352 and 3540 is 12.
- b. The required greatest number is 12.

Q14. (C)

3.36	1	4.32
3.36		3.5
.96		3.36
3.36		
x		

$$0.96 + 2 = 0.48 \text{ m}$$

Q15. (A)

5	25 - 40 - 120
5	5 - 8 - 24
8	1 - 8 - 24
3	1 - 1 - 3
	1 - 1 - 1

$$= 5 \times 5 \times 8 \times 3 = 600 \text{ chairs}$$

Q16. (A)

3	6 - 9 - 15
2	2 - 3 - 5
3	1 - 3 - 5
5	1 - 1 - 5
	1 - 1 - 1

$$\text{L.C.M of 6, 9 and 15} = 3 \times 2 \times 3 \times 5$$

= 90

∴ The bells will toll after 90 minutes, it mean at 6 : 30.

Multiple Choice Questions (MCQs)

- Q1. If $50x = 0.11$, then value of x is:
 (A) 2.2 (B) 0.2
 (C) 0.002 (D) 0.0022
- Q2. $\frac{(30 \times 2) - 6 + (12 + 6)}{\frac{2}{3}} =$
 (A) $37\frac{1}{3}$ (B) 84
 (C) 68 (D) 78
- Q3. $\frac{1}{1 - \frac{1}{7}} =$
 (A) $\frac{7}{6}$ (B) $\frac{6}{7}$
 (C) $\frac{8}{6}$ (D) 7
- Q4. If 1 is added to the denominator of a fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator, the fraction becomes 1. What is the fraction?
 (A) $\frac{2}{3}$ (B) $\frac{3}{2}$
 (C) $\frac{1}{2}$ (D) $\frac{5}{2}$
- Q5. Which one of the following is the least number of four digits divisible by 71?
 (A) 994 (B) 1071
 (C) 1065 (D) None of these
- Q6. How many numbers up to 200 are divisible 2 and 3 both?
 (A) 28 (B) 33
 (C) 100 (D) 50
- Q7. How many numbers between 150 and 500 are divisible by 2, 3 and 7 together?
 (A) 9 (B) 7
 (C) 11 (D) 2
- Q8. The product of two numbers is 120. The sum of their squares is 289. The sum of two numbers is:
 (A) 19 (B) 17
 (C) 23 (D) None of these
- Q9. The sum of two numbers is 100 and their difference is 37. The difference of their squares is:
 (A) 3700 (B) 2700
 (C) 5200 (D) 1900
- Q10. If $42 ? 8$ is a multiple of 9, then the digit represented by ? is:
 (A) 1 (B) 4
 (C) 8 (D) 2
- Q11. The smallest number which was when subtracted from 43079 makes it exactly divisible by 9 is:
 (A) 5 (B) 4
 (C) 6 (D) 7
- Q12. If a number is subtracted from the square of its one-half, the result is 48. The square root of the number is:
 (A) 4 (B) 5
 (C) 6 (D) 8

Explanatory Answers

Q1. (D) Given that

$$50x = 0.11 \Rightarrow x = \frac{0.11}{50}$$

$$\Rightarrow x = \frac{11}{50} \times \frac{1}{100} = \frac{11}{5000}$$

$$\Rightarrow x = 0.0022$$

Hence, correct answer is choice D.

Q2. (B) Given that $\frac{(30 \times 2) - 6 + (12 + 6)}{\frac{2}{3}}$

$$= \frac{(60) - 6 + (2)}{\frac{2}{3}} = (60 - 6 + 2) \times \frac{3}{2}$$

$$= \frac{56 \times 3}{2} = 28 \times 3 = 84$$

PEMDAS

Correct answer is choice B.

Q3. (A) Given that $\frac{1}{1 - \frac{1}{7}}$

$$= \frac{1}{\frac{6}{7}}$$

$$= \frac{1 \times \frac{7}{6}}{\frac{6}{7} \times \frac{7}{6}} = \frac{7}{6}$$

$$\frac{1}{1 - \frac{1}{7}}$$

$$\frac{7-1}{7} = \frac{6}{7}$$

Correct answer is choice A.

Q4. (A) Let the required fraction be $\frac{p}{q}$. Then

$$\frac{p}{q+1} = \frac{1}{2} \Rightarrow 2p = q+1$$

$$\Rightarrow 2p - q = 1 \quad \dots(1)$$

and $\frac{p+1}{q} = 1 \Rightarrow p+1 = q$

$$\Rightarrow p - q = -1 \quad \dots(2)$$

Subtracting (2) from (1), we have

$$\begin{array}{r} 2p - q = 1 \\ + p - q = -1 \\ \hline p = 2 \end{array}$$

Substituting the value of p in equation (2), we have

$$p - q = -1 \Rightarrow 2 - q = -1 \Rightarrow -q = -3 \Rightarrow \boxed{q=3}$$

\therefore The fraction is $\frac{2}{3}$.

Correct answer is choice A.

Q5. (C)

$$\begin{array}{r} 14 \\ 71 \overline{) 1000} \\ \underline{- 71} \\ 290 \\ \underline{- 284} \\ 6 \end{array}$$

\therefore The least number = $1000 - 6 + 71 = 1065$

Hence, correct answer is choice C.

Q6. (B) The L.C.M of 2 and 3 is 6. When 200 is divided by the L.C.M of 2 and 3, the answer is 33.

Correct answer is choice B.

Q7. (A) The L.C.M of 2, 3 and 7 is: $2 \times 3 \times 7 = 42$. A number which is divisible by 2, 3 and 7 is also divisible by its L.C.M, i.e., 42.

Up to 100 numbers divisible by 42 are : 2

Up to 500 numbers divisible by 42 are : 11

Between 100 and 500, divisible by 42 = $11 - 2 = 9$

Hence, correct answer is choice A.

Q8. (C) Let the numbers be x and y . Then

$$\begin{aligned} (x + y)^2 &= x^2 + y^2 + 2xy \\ &= (x^2 + y^2) + 2xy \\ &= 289 + (2 \times 120) = 289 + 240 = 529 \end{aligned}$$

$$\Rightarrow x + y = \sqrt{529} = 23 \quad \Rightarrow x + y = 23$$

Correct answer is choice C.

Q9. (A) Let the numbers be x and y . Then

$$x + y = 100 \quad \dots(1) \quad \text{and} \quad x - y = 37 \quad \dots(2)$$

Adding (1) and (2), we have

$$2x = 137 \quad \Rightarrow x = 68.5 \quad \text{Put in (1)}$$

$$68.5 + y = 100 \quad \Rightarrow y = 100 - 68.5 \quad \Rightarrow y = 31.5$$

$$(68.5)^2 - (31.5)^2 = 3700$$

Correct answer is choice A.

Q10.(B) Replacing ? by x . Then sum of digits

$$= (4 + 2 + x + 8) = 14 + x$$

Now, the least value for x for which $14 + x$ is divisible by 9 is 4.

Hence, correct answer is choice B.

Q11.(A) On dividing 43079 by 9, we have

$$\begin{array}{r} 4786 \\ 9 \overline{) 43079} \\ \underline{36} \\ 70 \\ \underline{63} \\ 77 \\ \underline{72} \\ 59 \\ \underline{54} \\ 5 \end{array}$$

Remainder = 5

\therefore The required number to be subtracted = 5.

Correct answer is choice A.

Q12.(A) Let the number be x , then

$$\frac{x^2}{4} - x = 48$$

$$\Rightarrow x^2 - 4x = 192$$

$$\Rightarrow x^2 - 4x - 192 = 0$$

$$\Rightarrow (x - 16)(x + 12) = 0$$

$$\Rightarrow x = 16 \quad \text{or} \quad x = -12$$

$$\Rightarrow x = 16 \Rightarrow \sqrt{x} = \sqrt{16} \Rightarrow \sqrt{x} = 4$$

Hence, correct answer is choice A.

Square Root

Methods of Finding Square Root:

By Factors. Resolve the number into its prime factors. The square root is the product of the prime factors taken half as many times as they occur in the number.

For example, square root of 49 is 7 because

$$7^2 = 7 \times 7 = 49$$

The square root of a number is denoted by the symbol $\sqrt{\quad}$, called the radical sign. Thus

$$\sqrt{49} = 7, \quad \sqrt{81} = 9 \text{ and } \sqrt{64} = 8$$

Note: $\sqrt{1} = 1$

Methods of Finding the Square Root:

Finding square root by factorization:

1. Find the prime factors of the given number.
2. Group the factors in pairs.
3. Take one number from each pair of factors and then multiply together.

Example 1:

Find the square root of the following:

- (i) 52900 (ii) 4624

Solution:

(i) 52900
 $= 2 \times 2 \times 5 \times 5 \times 23 \times 23$

$\therefore \sqrt{52900} = 2 \times 5 \times 23 = 230$

(ii) 4624
 $= 2 \times 2 \times 2 \times 2 \times 17 \times 17$

$\therefore \sqrt{4624} = 2 \times 2 \times 17 = 68$

2	52900
2	26450
5	13225
5	2645
23	529
23	23
	1

2	4624
2	2312
2	1156
2	578
17	289
17	17
	1

Multiple Choice Questions (MCQs)

Q1. If $\frac{250}{\sqrt{x}} = 10$, then $x =$

- (A) 625 (B) 250
 (C) 25 (D) None of these

Q2. If $\frac{\sqrt{y}}{200} = 0.02$, then $y =$

- (A) 2 (B) 16
 (C) 4 (D) 49

Q3. The square root of .09 is:

- (A) 0.3 (B) 0.03
 (C) 0.003 (D) 3

Q4. What is the value of $\sqrt{0.0009} + \sqrt{0.01}$?

- (A) 13 (B) 1.3
 (C) 0.13 (D) 0.013

Q5. $\sqrt{10} \times \sqrt{15}$ equals:

- (A) $6\sqrt{5}$ (B) $3\sqrt{6}$
 (C) $3\sqrt{5}$ (D) $5\sqrt{6}$

Q6. $\frac{1}{\sqrt{9}-\sqrt{8}}$ equals:

- (A) $3-2\sqrt{2}$ (B) $3+2\sqrt{2}$
 (C) $2-2\sqrt{3}$ (D) $3-2\sqrt{3}$

Q7. What is the value of $\sqrt[3]{0.00027}$?

- (A) 3 (B) 0.03

- (C) 0.3 (D) 0.003
- Q8. After simplifying $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$, the answer is:
- (A) $1+\sqrt{15}$ (B) $\sqrt{1-15}$
 (C) $2-\sqrt{3}$ (D) $\sqrt{5}-\sqrt{3}$
- Q9. If $\sqrt{3} = 1.732$, then $\sqrt{12} =$
- (A) 6.928 (B) 3.732
 (C) 3.464 (D) 5.732
- Q10. A gardener plants 17956 trees in such a way that there are as many rows as there are trees in a row. The number of trees in a row are:
- (A) 136 (B) 134
 (C) 154 (D) 144

- Q11. If x and y are equal and $\sqrt{4\left(\frac{x^2}{3y}\right)} = 1$, then what must be true for the value of y ?
- (A) y must be negative (B) y must be positive
 (C) y must be equal to 4 (D) y may have any value

- Q12. What is the smallest integer greater than $\sqrt{99}$?
- (A) 3 (B) 9
 (C) 10 (D) 50

- Q13. If $r^3 = 343$, then $3r =$
- (A) 7 (B) 21
 (C) 49 (D) 39

- Q14. Which shows $\sqrt{45} + \sqrt{245} - \sqrt{320}$ simplified completely?
- (A) $-12\sqrt{5}$ (B) $-12\sqrt{2}$
 (C) $2\sqrt{5}$ (D) $5\sqrt{2}$

- Q15. Which shows $\sqrt{\frac{7}{12}}$ simplified completely?

- (A) $\frac{\sqrt{7}}{12}$ (B) $\frac{\sqrt{14}}{12}$
 (C) $\frac{\sqrt{21}}{12}$ (D) $\frac{\sqrt{21}}{6}$

- Q16. If $p = \frac{\sqrt{3}-2}{\sqrt{2}+1}$, then which one of the following equals $p - 4$?

- (A) $\sqrt{3}-2$ (B) $\sqrt{3}+2$
 (C) 2 (D) $-2\sqrt{2} + \sqrt{6} - \sqrt{3} - 2$

- Q17. $\frac{4(\sqrt{6}+\sqrt{2})}{\sqrt{6}-\sqrt{2}} - \frac{2+\sqrt{3}}{2-\sqrt{3}} =$

- (A) 1 (B) $\sqrt{6}-\sqrt{2}$
 (C) $\sqrt{6}+\sqrt{2}$ (D) 8

- Q18. A perfect square is a positive integer which when square rooted results in an integer. If $N = 3^4 \cdot 5^3 \cdot 7$, then what is the biggest perfect square that is a factor of N ?

- (A) 3^2 (B) 5^2
 (C) $(9.5)^2$ (D) $(3.5.7)^2$

- Q19. If $\sqrt{24} = 4.899$, then the value of $\sqrt{\frac{8}{3}}$ is:

- (A) 1.333 (B) 2.333
 (C) 1.633 (D) 3.633

- Q20. If $\sqrt{2} = 1.4142$, then $\left(\frac{4+\sqrt{2}}{\sqrt{2}+1}\right)$ is equal

- to:
- (A) 2.2426 (B) 3.5126
 (C) 2.3462 (D) None of these

- Q21. If $\sqrt{2} = 1.4142$, then $\frac{\sqrt{2}}{2+\sqrt{2}}$ is equal to:

- (A) 0.4142 (B) 2.4142
 (C) $\frac{1}{0.4142}$ (D) None of these

Explanatory Answers

- Q1. (A) Given that $\frac{250}{\sqrt{x}} = 10$
 $\Rightarrow 250 = 10 \times \sqrt{x}$
 $\Rightarrow (25)^2 = (\sqrt{x})^2$

$$\Rightarrow x = 625$$

Correct answer is choice A.

Q2. (B) Given that $\frac{\sqrt{y}}{200} = 0.02$

$$\Rightarrow \sqrt{y} = 0.02 \times 200$$

$$\Rightarrow \sqrt{y} = \frac{2}{100} \times 200$$

$$\Rightarrow \sqrt{y} = 2 \times 2 = 4$$

$$\Rightarrow (\sqrt{y})^2 = (4)^2$$

$$\Rightarrow \boxed{y = 16}$$

Correct answer is choice B.

Q3. (A) Given that $\sqrt{.09}$

$$\Rightarrow \sqrt{\frac{9}{100}} \Rightarrow \sqrt{\frac{(3)^2}{(10)^2}}$$

$$\Rightarrow \frac{3}{10} = 0.3$$

Correct answer is choice A.

Q4. (C) Given that $\sqrt{0.0009} + \sqrt{0.01}$

$$\Rightarrow \sqrt{\frac{9}{10000}} + \sqrt{\frac{1}{100}}$$

$$\Rightarrow \sqrt{\frac{(3)^2}{(100)^2}} + \sqrt{\frac{(1)^2}{(10)^2}}$$

$$\Rightarrow \frac{3}{100} + \frac{1}{10} \Rightarrow \frac{3+10}{100} = \frac{13}{100} = 0.13$$

Correct answer is choice C.

Q5. (D) Given that $\sqrt{10} \times \sqrt{15}$

$$= \sqrt{2 \times 5} \times \sqrt{3 \times 5}$$

$$= \sqrt{2 \times 3 \times 5 \times 5}$$

$$= \sqrt{2 \times 3 \times 5^2}$$

$$= 5\sqrt{6}$$

Correct answer is choice D.

Q6. (B) Given that $\frac{1}{\sqrt{9} - \sqrt{8}}$

$$= \frac{1}{\sqrt{(3)^2} - \sqrt{2 \times 2 \times 2}}$$

$$= \frac{1}{3 - 2\sqrt{2}}$$

Multiplying and dividing by $3 + 2\sqrt{2}$

$$= \frac{1}{(3-2\sqrt{2})} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}} = \frac{3+2\sqrt{2}}{9-4(2)} = \frac{3+2\sqrt{2}}{9-8}$$

$$= \frac{3+2\sqrt{2}}{1} = 3+2\sqrt{2}$$

Correct answer is choice B.

Q7. (B) Given that $\sqrt[3]{0.000027}$

$$= \left(\frac{27}{1000000} \right)^{1/3}$$

$$= \frac{(3^3)^{1/3}}{(10^6)^{1/3}} = \frac{3^{3 \times \frac{1}{3}}}{10^{6 \times \frac{1}{3}}}$$

$$= \frac{3}{10^2} = \frac{3}{100}$$

$$= 0.03$$

Correct answer is choice B.

Q8. (A) Given that $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Multiplying numerator and denominator by $\sqrt{5}-\sqrt{3}$, we have

$$\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2 - (\sqrt{3})^2} = \frac{5-3+2\sqrt{15}}{5-3}$$

$$= \frac{2+2\sqrt{15}}{2} = \frac{2(1+\sqrt{15})}{2}$$

$$= 1+\sqrt{15}$$

Correct answer is choice A.

Q9. (C) Given that $\sqrt{3} = 1.732$

As $\sqrt{12} = \sqrt{2 \times 2 \times 3}$

$$= 2\sqrt{3}$$

$$= 2 \times 1.732 \quad (\sqrt{3} = 1.732 \text{ given})$$

$$= 3.464$$

Correct answer is choice C.

Q10.(B) Number of trees in a row = $\sqrt{17956}$
= 134

Correct answer is choice B.

Q11.(B) Because a negative number cannot have a real square root, the value under a square root sign must be positive. Thus, correct answer is choice B.

Q12.(C) Given that $\sqrt{99}$. Evaluating

$$\sqrt{99} = 9.9498$$

Clearly 10 is the smallest unit greater than the square root of 99. As $10 > 9.9498$.

Correct answer is choice C.

Q13.(B) Given that

$$\begin{aligned} r^3 &= 343 \\ r^3 &= (7)^3 \\ (r^3)^{1/3} &= (7)^{3 \times 1/3} \\ r &= 7 \\ \Rightarrow \boxed{3r} &= \boxed{21} \end{aligned}$$

Q14.(C) Given that $\sqrt{45} + \sqrt{245} - \sqrt{320}$

$$\begin{aligned} &= \sqrt{3 \times 3 \times 5} + \sqrt{7 \times 7 \times 5} - \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 5} \\ &= 3\sqrt{5} + 7\sqrt{5} - 2 \times 2 \times 2\sqrt{5} \\ &= 3\sqrt{5} + 7\sqrt{5} - 8\sqrt{5} \\ &= \sqrt{5}(3 + 7 - 8) \\ &= 2\sqrt{5} \end{aligned}$$

Correct answer is choice C.

Q15.(D) Given that $\sqrt{\frac{7}{12}}$. We can write it as $\frac{\sqrt{7}}{\sqrt{12}}$.

Rationalization

$$\begin{aligned} &= \frac{\sqrt{7}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} = \frac{\sqrt{7 \times 12}}{\sqrt{12 \times 12}} \\ &= \frac{\sqrt{7 \times 3 \times 2 \times 2}}{\sqrt{12 \times 12}} = \frac{2\sqrt{21}}{12} \\ &= \frac{\sqrt{21}}{6} \end{aligned}$$

5	245
7	49
	7
2	320
2	160
2	80
2	40
2	20
2	10
	5

Correct answer is choice D.

Q16.(D) Given that $p = \frac{\sqrt{3}-2}{\sqrt{2}+1}$

Rationalization

$$\begin{aligned} &\frac{\sqrt{3}-2}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} \\ &= \frac{\sqrt{3 \times 2} - 2\sqrt{2} - \sqrt{3} + 2}{(\sqrt{2})^2 - (1)^2} \\ &= \frac{\sqrt{6} - 2\sqrt{2} - \sqrt{3} + 2}{2-1} = \sqrt{6} - 2\sqrt{2} - \sqrt{3} + 2 \end{aligned}$$

Now

$$\begin{aligned} p - 4 &= \sqrt{6} - 2\sqrt{2} - \sqrt{3} + 2 - 4 \\ &= \sqrt{6} - 2\sqrt{2} - \sqrt{3} - 2 \\ &= -2\sqrt{2} + \sqrt{6} - \sqrt{3} - 2 \end{aligned}$$

Correct answer is choice D.

Q17.(A) Given that $\frac{4(\sqrt{6} + \sqrt{2})}{\sqrt{6} - \sqrt{2}} - \frac{2 + \sqrt{3}}{2 - \sqrt{3}}$

Let's rationalize both fractions by multiplying top and bottom of each fraction by its conjugate of its denominator.

$$\begin{aligned} \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} &= \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}} \times \frac{\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}} \\ &= \frac{(\sqrt{6}+\sqrt{2})^2}{(\sqrt{6})^2 - (\sqrt{2})^2} \\ &= \frac{6+2+4\sqrt{3}}{6-2} = \frac{8+4\sqrt{3}}{4} = \frac{4(2+\sqrt{3})}{4} \\ &= 2+\sqrt{3} \end{aligned}$$

Thus $\frac{4(\sqrt{6}+\sqrt{2})}{\sqrt{6}-\sqrt{2}} = 4(2+\sqrt{3}) = 8+4\sqrt{3}$

Now $\frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$$\begin{aligned} &= \frac{(2+\sqrt{3})^2}{(2)^2 - (\sqrt{3})^2} = \frac{4+3+4\sqrt{3}}{4-3} \\ &= \frac{7+4\sqrt{3}}{1} = 7+4\sqrt{3} \end{aligned}$$

Hence, $\frac{4(\sqrt{6}+\sqrt{2})}{\sqrt{6}-\sqrt{2}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} = 8+4\sqrt{3} - 7 - 4\sqrt{3} = 1$

Correct answer is choice A.

Q18.(C) Here, Given that $N = 3^4 \cdot 5^3 \cdot 7$

Writing N as a product of perfect square yields $(3^4 \cdot 5^2) \cdot 5 \cdot 7 = (3^2 \cdot 5)^2 \cdot 5 \cdot 7 = (9 \cdot 5)^2 \cdot 5 \cdot 7$.
Hence, $(9 \cdot 5)^2$ is the biggest perfect square factor of N.

The correct choice is C.

Q19.(C) Given that $\sqrt{24} = 4.899$

$$\begin{aligned} \frac{\sqrt{8}}{\sqrt{3}} &= \frac{\sqrt{8}}{\sqrt{3}} \\ &= \frac{\sqrt{8}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{\sqrt{24}}{\sqrt{3 \times 3}} \\ &= \frac{\sqrt{24}}{3} = \frac{4.899}{3} \\ &= 1.633 \end{aligned}$$

Hence, the correct answer is C.

Q20.(A) Given that $\frac{4+\sqrt{2}}{\sqrt{2}+1}$

$\therefore \frac{4+\sqrt{2}}{\sqrt{2}+1} = \frac{4+\sqrt{2}}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$$\begin{aligned} &= \frac{(4+\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2})^2 - (1)^2} = \frac{4\sqrt{2}+2-4-\sqrt{2}}{2-1} \end{aligned}$$

$$\begin{aligned}
 &= \frac{4\sqrt{2} - \sqrt{2} - 2}{1} = 3\sqrt{2} - 2 \\
 &= 3(1.4142) - 2 \\
 &= 4.2426 - 2 \\
 &= 2.2426
 \end{aligned}$$

Correct answer is choice A.

Q21.(A) Given that $\frac{\sqrt{2}}{2+\sqrt{2}}$

$$\begin{aligned}
 \therefore \frac{\sqrt{2}}{2+\sqrt{2}} &= \frac{\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} \\
 &= \frac{2\sqrt{2} - \sqrt{2} \times 2}{(2)^2 - (\sqrt{2})^2} = \frac{2\sqrt{2} - 2}{4-2} = \frac{2\sqrt{2} - 2}{2} \\
 &= \frac{2(\sqrt{2} - 1)}{2} = \sqrt{2} - 1 \\
 &= 1.4142 - 1 = 0.4142
 \end{aligned}$$

Correct answer is choice A.

Fractions & Decimals

FRACTIONS:

If any unit be divided into any number of equal parts, one or more of these parts is called a fraction of the unit.

Example: The fractions one-fourth, two-third and three-fourth are respectively written as $\frac{1}{4}$, $\frac{2}{3}$ and $\frac{3}{4}$.

NUMERATOR AND DENOMINATOR:

The upper number, which shows the number of parts taken to form the fraction, is called numerator.

The lower number, which indicates the number of equal parts in which the unit is divided, is called denominator.

Terms of the Fraction:

The numerator and the denominator of a fractions are called its terms.

Note: A fraction is also called a rational number.

Lowest Terms of a Fraction:

When the numerator and the denominators of a fraction have no common factor, the fraction is said to be in its lowest terms:

$$\text{Example: } = \frac{6}{10} = \frac{3 \times 2}{5 \times 2}$$

In the above example, denominator and the numerator have a common factor, thus $\frac{6}{10}$ is not in its lowest terms. If we cancel out 2 by dividing numerator and denominator by 2, we find $\frac{3}{5}$, which has no common factor. Hence $\frac{3}{5}$ is in its lowest terms.

Proper Fraction:

A proper fraction is one whose numerator is less than the denominator.

Example: $\frac{2}{3}, \frac{5}{7}, \frac{23}{46}$ are proper fractions.

Note: The value of proper fractions is always less than 1.

Multiple Choice Questions (MCQs)

- Q1. If $\frac{5}{x}, \frac{8}{x}$ and $\frac{13}{x}$ are all in lowest terms. Then how many integers, x , between 30 and 40?
- (A) 5 (B) 1
(C) 2 (D) 3
(D) None of these
- Q2. $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$ equals:
- (A) $\frac{1}{120}$ (B) $\frac{1}{2}$
(C) $\frac{1}{30}$ (D) 1
(D) None of these
- Q3. If $\frac{4}{13}$ of a number is 39, what is $\frac{8}{13}$ of that number?
- (A) $\frac{39}{4}$ (B) 78
(C) 16 (D) $\frac{39}{8}$
- Q4. $\frac{3}{4}$ of 28 is equal to $\frac{7}{30}$ of what number?
- (A) 90 (B) 45
(C) 30 (D) 56
- Q5. Which of the following is less than $\frac{5}{11}$?
- (A) $\frac{3}{2}$ (B) $\frac{2}{3}$
(C) $\frac{1}{2}$ (D) $\frac{2}{5}$
- Q6. There are 20 boys in a class. Five of them are left-handed. What fraction of the class is left-handed?
- (A) $\frac{1}{5}$ (B) $\frac{1}{2}$
(C) $\frac{1}{4}$ (D) $\frac{2}{11}$
- Q7. A chemical solution contains 8% of acid. If there is 15 ml of acid, what is the volume of the solution?
- (A) 125.5 mL (B) 187.5 mL
(C) 225.5 mL (D) 171.5 mL
- Q8. What fractional part of a week is 98 hours?
- (A) $\frac{7}{98}$ (B) $\frac{7}{12}$
(C) $\frac{1}{20}$ (D) $\frac{1}{7}$
- Q9. A village has 5860 voters, of whom 7% usually forget to vote. In order to win an election, a candidate must gain at least 50% of the remaining votes. How many votes does he need in order to win?
- (A) 2725 (B) 410
(C) 5450 (D) None of these
- Q10. What fraction is exactly midway between $\frac{1}{3}$ and $\frac{1}{4}$?
- (A) $\frac{7}{12}$ (B) $\frac{7}{24}$
(C) $\frac{29}{11}$ (D) $\frac{1}{2}$
- Q11. $\frac{4}{9}$ of a number is 12. What is the number?
- (A) 27 (B) 36
(C) 18 (D) 16
- Q12. Ali purchased some goldfish. During the first week, $\frac{1}{5}$ of them died, and during the second week, $\frac{3}{8}$ of those still alive at the end of the first week died. What is the fraction of the original goldfish still alive after two weeks?
- (A) $\frac{1}{2}$ (B) $\frac{3}{2}$
(C) $\frac{5}{2}$ (D) $\frac{4}{3}$
- Q13. $\frac{3}{8}$ of a number is 10. What is the number?
- (A) 91 (B) 81
(C) 23 (D) 27
- Q14. $\frac{5}{8}$ of 24 is equal to $\frac{15}{7}$ of what number?
- (A) 15 (B) 105

- (C) 35 (D) 7
 Q15. A German class has 12 boys and 18 girls. What is the fraction of the class boys? (C) $\frac{2}{3}$ (D) $\frac{2}{5}$
 (A) $\frac{1}{6}$ (B) $\frac{3}{5}$

Explanatory Answers

Q1. (D) If x is even, then $\frac{8}{x}$ will not be in lowest term. This is because, both x and 8 are divisible by 2. Now we take the odd number between 30 and 40, these are 31, 33, 35, 37, 39. In these numbers, we see that 35 and 39 are divisible by 5 and 13, respectively. Thus only 31, 33 and 37 are required numbers.

Q2. (A) Simplifying $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$
 $\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$

Q3. (B) As $\frac{4}{13}$ of a number is 39. Therefore, the $\frac{8}{13}$ of that number will be 78.

Because $\frac{8}{13} = \frac{4}{13} \times 2$, and $\frac{4}{13}$ of a number is 39, therefore, double of $\frac{4}{13}$ ($\frac{4}{13} \times 2 = \frac{8}{13}$) should be equal to $39 \times 2 = 78$.

Q4. (A) Let x be the required number, then by given condition

$$28 \div \frac{4}{3} = x \div \frac{30}{7}$$

$$28 \times \frac{3}{4} = x \times \frac{7}{30}$$

$$21 = x \times \frac{7}{30}$$

$$\frac{21 \times 30}{7} = x$$

$$\Rightarrow \boxed{x = 90}$$

Q5. (D)

Q6. (C) Left-handed = 5
 Total = 20

So, fraction = $\frac{5}{20} = \frac{1}{4}$

Q7. (B) 8 mL acid in solution = 100 mL
 1 mL acid in solution = $\frac{100}{8} = 12.5$ mL
 15 mL acid in solution = $12.5 \times 15 = 187.5$ mL

Q8. (B) There are 7 days in a week, and each day has 24 hours. Therefore, hours in a week = $24 \times 7 = 168$

The required fraction is: $\frac{98}{168} = \frac{7}{12}$

Q9. (A) People does not give vote = $\frac{7}{100} \times 5860$

$$= 7 \times 58.6$$

$$= 410.2$$

People does not give vote \cong 410 people

$$\text{Remaining people} = 5860 - 410$$

$$= 5450 \text{ people}$$

$$\text{Candidate must gain vote} = 5450 \times \frac{50}{100}$$

$$= 2725 \text{ vote}$$

Q10. (B) The midway fraction of the fractions $\frac{1}{3}$ and $\frac{1}{4} = \frac{1}{2} \left(\frac{1}{3} + \frac{1}{4} \right) = \frac{1}{2} \left(\frac{7}{12} \right) = \frac{7}{24}$.

Q11. (A) Let the required number be "x", then according to given condition $\frac{4}{9} \times x = 12 \Rightarrow x = \frac{12}{\frac{4}{9}}$

$$= 12 \times \frac{9}{4} = 27$$

Q12. (A) Let the number of fish purchased $= x$

$$\text{During first week (died fish)} = \frac{1}{5} \times x = \frac{x}{5}$$

$$\text{Still alive} = x - \frac{1}{5}x = \frac{4}{5}x$$

$$\text{During second week (died fish)} = \frac{4}{5}x \times \frac{3}{8} = \frac{3}{10}x$$

$$\text{Fish at the end of two weeks} = \frac{4x}{5} - \frac{3x}{10} = \frac{8x - 3x}{10} = \frac{5x}{10} = \frac{1}{2}x$$

$$\text{So fraction} = \frac{\frac{1}{2}x}{x} = \frac{1}{2}$$

Q13. (D) Let the number $= x$

$$\text{Then } \frac{3}{8} \times x = 10$$

$$\Rightarrow x = \frac{80}{3}$$

$$\Rightarrow x = 26.67 = \boxed{27}$$

Q14. (D) Let the number $= x$

$$\text{Then } \frac{15}{7} \times x = \frac{5}{8} \times 24$$

$$\Rightarrow \frac{15 \times x}{7} = 15$$

$$\Rightarrow x = \frac{7 \times 15}{15} = \boxed{7}$$

Q15. (D) No. of boys = 12

No. of girls = 18

$$\text{Total} = 12 + 18 = 30$$

$$\text{Required fraction} = \frac{12}{30} = \boxed{\frac{2}{5}}$$

Percentage

Percentage:

The term 'percent' is a short form of the Latin word 'per centum' meaning 'out of hundred'. It can best be defined as:

"A fraction whose denominator is 100 is called a percentage and the numerator of the fraction is called the rate percent."

A rate percent is reduced to an equivalent fraction dividing it by 100.

Change of percentage into Fraction or Decimal:

To convert a percentage to a fraction, mixed number or decimal, divide it by 100, and reduce, if possible. If necessary, the relating fraction may then be changed to a decimal.

Example 1:

(i) Express $2\frac{1}{7}\%$ to a fraction

(ii) Change $\frac{3}{4}\%$ to a decimal.

Solution:

$$\begin{aligned} \text{(i)} \quad 2\frac{1}{7}\% &= \frac{15}{7}\% \\ &= \frac{15}{7} \times \frac{1}{100} \left(\text{Replace \% by } \frac{1}{100} \right) \\ &= \frac{3}{140} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad \frac{3}{4}\% &= \frac{3}{4} \times \frac{1}{100} \left(\text{Replace \% by } \frac{1}{100} \right) \\ &= \frac{3}{400} = .0075 \end{aligned}$$

Multiple Choice Questions (MCQs)

Q1. If the base of a rectangle is increased by 40% and its altitude is decreased by 20%, then its area is:

- (A) decreased by 20% (B) increased by 12%
(C) decreased by 12% (D) increased by 16%

Q2. If $x\%$ of y is 20, then $y =$

- (A) $2000x$ (B) $\frac{100}{x}$
(C) $\frac{2000}{x}$ (D) $\frac{x}{200}$

Q3. 12 is $\frac{1}{3}\%$ of what number?

- (A) 4 (B) 400
(C) 36 (D) 3600

Q4. If p is a positive number, 400% of p is what percent of 400 p ?

- (A) 4 (B) 25
(C) 40 (D) 1

Q5. What is 10% of 30% of 40%?

- (A) 0.12% (B) 0.012%
(C) 12% (D) 1.2%

Q6. What percent of 75 is x ?

- (A) $\frac{3}{4}x$ (B) $\frac{4}{3}x$
(C) $4x$ (D) $3x$

Q7. If 35 students took an exam and 13 of

- them failed, what percent of them passed?
 (A) 20% approx (B) 63% approx
 (C) 25% approx (D) 22% approx
- Q8. There are twice as many boys as girls in an economics class. If 20% of the boys and 35% of the girls have already handed over their result cards, what percent of the students have not yet handed over their cards?
 (A) 75 (B) 65
 (C) 55 (D) 15
- Q9. A dealer bought an ornamental jar for Rs. 7,000 and after some days sold it for Rs. 21,000. By what percent did the value of jar increase?
 (A) 300 (B) 200
 (C) 150 (D) 20
- Q10. On a test consisting of 60 problems, Sonia solved 75% of first 40 problems correctly. What percent of the other 20 questions does she need to solve correctly for her grade on the entire exam to be 90%?
 (A) 95% (B) 65%
 (C) 85% (D) cannot achieve 90%
- Q11. If 60% of A is 30% of B, then B is what percent of A?
 (A) 300% (B) 30%
 (C) 200% (D) 3%
- Q12. What percent of p is q ?
 (A) $\frac{q}{p}$ (B) $\frac{q}{p}$
 (C) $\frac{100q}{p}$ (D) $\frac{100p}{q}$
- Q13. What percent of $\frac{1}{2}$ is $\frac{5}{4}$?
 (A) 2.5 (B) 1.5
 (C) 250 (D) 150
- Q14. In a school of 820 students, 55% are boys. The number of girls and the number of boys are:
 (A) 369 boys, 451 girls (B) 281 boys, 539 girls
 (C) 539 boys, 281 girls (D) 451 boys, 369 girls
- Q15. Jafer drew a square. He then erased it and drew a second square whose sides were 3 times the sides of the first square. By what percent was the area of the square increased?
 (A) 300% (B) 800%
 (C) 400% (D) 200%
- Q16. A team has won 60 percent of the 20 games for all this season. If the team plays a total 50 games all season and wins 80 percent of the remaining games, how many games will the team win for the entire season?
 (A) 36 (B) 25
 (C) 42 (D) 39
- Q17. Local telephone calls increased in price from 25 Pa to 30 Pa. What percentage increase was this?
 (A) 15% (B) 25%
 (C) 5% (D) 20%
- Q18. A worker pays Rs. 350 tax per month, which is 15% of his income. What is his income?
 (A) 3500 (B) 5250
 (C) 2333.33 (D) 2523.30
- Q19. If "x" is a positive number, 400% of x is what percent of 400x?
 (A) 1 (B) 0.1
 (C) 0.01 (D) 100
- Q20. Babar gave 15% of his baseball cards to Laeeq and 20% to Sarfraz. If he still had 520 cards, how many did he have originally?
 (A) 800 (B) 720
 (C) 820 (D) 600
- Q21. A certain country has an infant mortality rate of 6.8% of 20000 babies born in a certain year, how many survived?
 (A) 1360 (B) 18640
 (C) 18000 (D) 17640
- Q22. 20% of 50% of 80 is:
 (A) 40 (B) 16
 (C) 8 (D) 60
- Q23. The price of a can of acid was increased by 20%. How many cans can be purchased for the amount of money that used to buy 300 cans?
 (A) 250 (B) 320
 (C) 150 (D) 240
- Q24. In a basket containing 180 pears, 9 pears are spoiled. What percent of the pears in the basket are not spoiled?
 (A) 85% (B) 5%
 (C) 95% (D) 9%

Q25. A silo (container for storing grain) is filled to capacity with p kilograms of wheat. Rats eat q kilograms a day. After 21 days, what percentage of the silo's capacity have the rats eaten?

(A) $\frac{21q}{30p} \times 100$ (B) $\frac{q}{p} \times 100$

(C) $\frac{21q-p}{p} \times 100$ (D) $2100\left(\frac{q}{p}\right)$

Q26. A factory normally employs 100 people. During a slow spell, it fired 20% of its employees. By what percentage must it now increase its staff to return to full capacity?

(A) 25% (B) 20%
(C) 80% (D) 40%

Q27. Six students in a class failed in geometry. This represents $16\frac{2}{3}\%$ of the class. How many students passed the course?

(A) 36 (B) 30

(C) 42 (D) 24

Q28. If 30% of all women are voters and 42% of the population are women, what percent of the population are women voters?

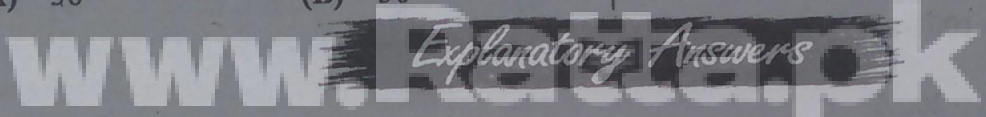
(A) 17.4% (B) 25.20%
(C) 12.60% (D) None of these

Q29. If the length of the rectangle is increased by 16% and the width is decreased by 25%, then the area:

(A) increases by 9% (B) decreases by 41%
(C) decreases by 13% (D) increases by 59%

Q30. If the base of a rectangle is increased by 40% and the altitude is decreased by 30%, the area is:

(A) increased by 10% (B) increased by 12%
(C) decreased by 10% (D) decreased by 2%



Q1. (B) If the value firstly increased by $x\%$ and then decreased by $y\%$ then there is $\left[x - y - \frac{xy}{100}\right]\%$ increase or decrease according as the sign +ve or -ve, respectively. In this problem, $x = 40$ and $y = 20$. Therefore,

$$\left[40 - 20 - \frac{(40)(20)}{100}\right]\%$$

$$\left[20 - \frac{800}{100}\right]\%$$

$$[20 - 8]\% = 12\%$$

Because sign is +ve, therefore, its area is increased by 12%.

Q2. (C) $y \times \frac{x}{100} = 20$

$$\Rightarrow xy = 20 \times 100 \Rightarrow xy = 2000$$

$$\Rightarrow y = \frac{2000}{x}$$

Q3. (D) Using, $\frac{\text{Part}}{\text{Whole}} = Y$ percent, here $P = 12$, $W = ?$ and Y percent = $\frac{1}{300}$

$$\frac{P}{W} = \frac{Y}{100} \Rightarrow \frac{P}{W} = Y \times \frac{1}{100}$$

$$\frac{12}{W} = \frac{1}{3} \times \frac{1}{100} \Rightarrow W = 3 \times 1200 = 3600$$

Q4. (D) 400% of $p = \frac{400}{100} \times p = 4p$, which is 1% of $400p$.

Q5. (D) $30\% \text{ of } 40\% = \frac{30}{100} \times \frac{40}{100} = \frac{12}{100} = 0.12$

Now $10\% \text{ of } 30\% \text{ of } 40\% = \frac{10}{100} \times 0.12 = 0.012 = 1.2\%$

Q6. (B) $\frac{P}{W} = \frac{Y}{100} \Rightarrow x = \frac{Y}{100} \times 75$

$\Rightarrow x = \frac{3Y}{4} \Rightarrow Y = \frac{4x}{3}$

Q7. (B) If 13 students failed, then the number of passed students = $35 - 13 = 22$

Thus, $\frac{22}{35} \times 100 = 63\% \text{ approx.}$

Q8. (A) Let the number of girls = 100, then

Number of boys = 200

Then 35 girls (35% of 100) and 40 boys (20% of 200), have handed in their cards. Hence 75 of the 300 (100 + 200) students have handed them in. It means that $300 - 75 = 225$ have not handed them in. Thus

$\frac{225}{300} \times 100 = 75\%$

Q9. (B) The increment in the value of the jar = Rs. 21000 - Rs. 7000 = Rs. 14000

The %age increase in the value of the jar

$= \frac{\text{Increment}}{\text{Actual}} \times 100$

$= \frac{14000}{7000} \times 100 = 200\%$

Q10.(D) To achieve 90% grade on the entire examination, Sonia needs 54 (as calculated below) problems:

$$\frac{P}{W} = y\% \Rightarrow \frac{P}{60} = \frac{90}{100} \Rightarrow P = \frac{90}{100} \times 60$$

$$\Rightarrow P = 54$$

to solve correctly. So far she has solved 30 $\left(\frac{P}{40} = \frac{75}{100} \Rightarrow P = \frac{75}{100} \times 40 = 30 \right)$ problems

correctly. Therefore, on the last 20 problems she needed $54 - 30 = 24$ correct answers, which is impossible to get from 20 problems.

Q11.(C) 60% of A is 30% of B, i.e., $\frac{60}{100}A = \frac{30}{100}B$.

$\Rightarrow .60A = .30B, \Rightarrow B = \frac{.60}{.30}A \Rightarrow B = 2A$

Now we find B is what percent of A. i.e.,

$B = \frac{x}{100}A \text{ or } B = (x\%)(A)$

$\Rightarrow B = (200\%)A$

Q12.(A) Using the relation $\frac{\text{Part}}{\text{Whole}} = y\%$

$\frac{q}{p} = y\%$

Second Method: What % p is q
 x % $p = q$

$$\Rightarrow x\% = \frac{q}{p}$$

Q13.(C) Using $\frac{\text{Part}}{\text{Whole}} = y\%$

$$\frac{5}{4} + \frac{1}{2} = y\%$$

$$\frac{5}{4} \times 2 = y\% \Rightarrow y\% = \frac{5}{2} = 2.5$$

$$\Rightarrow y\% = 250\%$$

Q14. (D) Total No. of students = 820

$$\text{No. of boys} = 820 \times \frac{55}{100} = 451 \text{ boys}$$

$$\text{No. of girls} = 820 - 451 = 369 \text{ girls}$$

Q15. (B)

Let the length of first square = 1 inch

Then Area of first square = 1 square inch

Then sides of the second square = 3 inch

Area of the second square = 9 square inch

\therefore Increase in the area of the 2nd square = 8 square inches

%age increase in the second square = $\frac{8}{1} \times 100 = 800\%$

Q16. (A) Total No. of games that

$$\text{the team has won so far} = \frac{60}{100} \times 20 = 12 \text{ games}$$

$$\text{The total number of games left} = 50 - 20 = 30$$

80% of 30 games will the team win

$$\therefore \frac{80}{100} \times 30 = 24 \text{ games}$$

$$\text{The total number of wins} = 12 + 24 = \boxed{36}$$

Q17. (D) Increase in local call = 30 - 25 = 5Pa

$$\% \text{ increase} = \frac{5}{25} \times 100 = 20\%$$

Q18. (C) Let "x" be his income then

$$15\% \text{ of } x = 350$$

$$x = 350 \div 15\% = 350 \times \frac{100}{15}$$

$$x = 2333.33$$

Q19. (A) 400% of x = 4x. Which is 1% of 400x.

Q20. (A) Actually, Babar had 100% of the cards. After distributing 35% (20% + 15%) of them, he had

100% - 35% = 65% of them left. So

$$520 = \frac{65}{100}x \Rightarrow x = \frac{520 \times 100}{65} = \boxed{800}$$

Q21. (B) Infant mortality = $20000 \times \frac{6.8}{100} = 1360$

$$\text{survived bodies} = 20000 - 1360 = \boxed{18640}$$

Q22. (C) $50\% \text{ of } 80 = 80 \times \frac{50}{100} = 40$

$20\% \text{ of } 40 = 40 \times \frac{20}{100} = \boxed{8}$

Q23. (A) Let the can of acid used to cost Rs. 1

After increasing 20% cost, it became $= 1 + \frac{1}{20} = 1.20$

Then 300 cans of acid used to cost = Rs. 300

Each can be bought for Rs 300

$\therefore 300 \div 1.20 = \boxed{250}$

Q24. (C) The pears that are not spoiled = $180 - 9 = 171$

Percentage = $\frac{171}{180} \times 100$

$= 19 \times 5$

$= 95\%$

Q25. (D) After 21 days the rats have eaten wheat = 21q kilograms.

So, the required fraction in percentage = $\frac{21q}{p} \times 100$

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Q26. (A) $20\% \text{ of } 100 = 20$ employee

employees left = $100 - 20 = 80$ employees

If it again increases by 20, the percentage of increase

$= \frac{20}{80} \times 100 = 25\%$

Q27. (B) Let x be the number of students, then

$16\frac{2}{3}\% = \frac{1}{6} \Rightarrow \frac{1}{6}x = 6$

$\Rightarrow x = 36$

36 students in class, 6 failed, 30 passed

Q28. (C) 30% of the 42% of the population who are women are voters so

$(.30)(.42) = 0.126 = 12.60\%$ of the population are women voters.

Q29. (C) Let L be the original length and W be the original width.

The new length = $100\% + 16\% = 116\%$ of L $\Rightarrow \boxed{1.16L}$

Since the width decreases by 25% so the new width is 75% of W $\Rightarrow .75W$

Area = LW

\Rightarrow New Area = $(1.16)(.75)LW$

$= 0.87LW = 87\%$ of Area

Since the area is 87% of the original area. Thus the area has decreased by $(100 - 87) = 13\%$

Q30. (D) Let "b" be the base and "a" altitude. Then the new base will be $(b + 0.4b)$. The new altitude after decreasing 30% is $(a - 0.3a)$.

So the area is

$((a - .3a)(b + 0.4b) = (0.7)(1.4)ab$
 $= 0.98ab$

The new area is 98% of the old. So the new area (98% - 100%)

= -2% is decreased by 2%

Ratio & Proportion

RATIO:

The number of times one quantity contains another quantity of the same kind is called the ratio of the two quantities.

Note: The ratio of two quantities is equivalent to the fraction that one quantity is to the other.

Example: There can be ratio between Rs. 30 and Rs. 40, but there can be no ratio between Rs. 30 and 40 apples.

Remember: The ratio 3:5 is written as 3:5 or $\frac{3}{5}$, 3 and 5 are called the terms of the ratio. 3 is the first and 5 is the second term.

Note: The first term of a ratio is called the antecedent and the second the consequent.

If a set of objects is divided into two groups in the ratio $a : b$, then the first group contains $\frac{a}{a+b}$ of the total objects. The second group contain $\frac{b}{a+b}$ of the total number of objects.

Important Example:

If a bag containing twelve mirrors is dropped, which of the following cannot be the ratio of the broken mirrors to unbroken mirrors?

- (i) 2:1 ii) 3:1 iii) 3:2 iv) 1:1 v) 7:5

Solution:

Since there are 12 mirrors in the bag. So 12 must be divisible by the sum of terms in the ratio exactly. We see that $2+1=3$ divides 12 exactly $3+1=4$ also divides exactly. Only the ratio $3+2=5$ doesn't divide 12 exactly. Thus the correct answer is (iii).

PROPORTION:

The equality of ratios is called proportions.

Example:

Consider the two ratios

1st ratio	2nd ratio
5:15	7:21

Since 5 is one-third of 15, and 7 is one-third of 21, the two ratios are equal.

Note: The first and fourth terms are called extremes, and the second and third terms are called the means. In above example, 5 and 21 are extremes, while 15 and 7 are means.

Multiple Choice Questions (MCQs)

Q1. In a city 90% of the population own a car, 15% own a motorcycle, and everybody owns one or the other or both. What is the percentage of motorcycle owners to who own cars?

- (A) 15% (B) 5%
(C) 75% (D) $33\frac{1}{3}\%$

Q2. Concrete consists of cement, sand and

screenings in the ratio of 1 : 5 : 4, what is the percentage of the sand mixed?

- (A) 10% (B) 40%
(C) 50% (D) 60%

Q3. Three business partners shares have profit of Rs. 24000 in the ratio 5 : 4 : 3. What is the amount of the least share?

- (A) 6000 (B) 8000
(C) 10,000 (D) 1200

Q4. A machine produces 1280 parts in 16 hours. How many parts would it make in a working week of 44 hours?

- (A) 2530 (B) 3520
(C) 2122 (D) 3960

Q5. If the ratio of x and y is $\frac{11}{3}$, what is the value of $2x$ to y ?

- (A) $\frac{11}{6}$ (B) $\frac{22}{6}$
(C) $\frac{22}{3}$ (D) $\frac{11}{5}$

Q6. If 80% application to a program were rejected, what is the ratio of the number accepted to the number rejected?

- (A) 1 : 4 (B) 4 : 1
(C) 1 : 8 (D) 3 : 8

Q7. What is the ratio of the circumference of a circle to its radius?

- (A) π (B) $\frac{\pi}{2}$
(C) 2π (D) 2π

Q8. Win/Loss ratio for two teams are A, 5 : 2 and B, 7 : 3 which team has the better record?

- (A) A (B) B
(C) both A and B (D) wrong question

Q9. If 15 workers can paint a certain number of houses in 24 days, how many days will 40 workers take, working at the same rate, to do the same job?

- (A) 12 days (B) 18 days
(C) 15 days (D) 9 days

Q10. If a jet travels 1280 km in 2 hours, how far will it travel in $5\frac{1}{2}$ hours, at the same speed?

- (A) 2100 (B) 3300
(C) 2700 (D) 3520

Q11. If the ratio of $a : b$ is 9 : 7 then $a + b$ is:

- (A) 14 (B) 16
(C) 63 (D) not possible

Q12. If you can buy A apples for n nickels (five cent coin), how many apples can you buy for d dimes and q quarters?

- (A) $\frac{A(d+q)}{n}$ (B) $\frac{A}{n}(10d+25q)$
(C) $\frac{A}{n}(2d+5q)$ (D) $\frac{d+q}{An}$

Q13. If the ratio of boys and girls in a class is 3 : 5 and the class contains 24 students, how many additional boys would have to enroll to make the ratio of boys to girls 1:1?

- (A) 9 (B) 15
(C) 6 (D) 12

Q14. A recipe requires 13 gram of sugar and 18 gram of flour. If only 100 gram of sugar is used, how much flour, to the nearest gram, should be used?

- (A) 167.3 (B) 138.13
(C) 144.5 (D) 178.12

Q15. Green paint is obtained from blue and yellow paint in the ratio 3 : 5. How much of each colour is needed to make 40 litres of this green paint?

- (A) Blue paint: 15 litres, yellow paint: 25 litres (B) Blue paint: 25 litres, yellow paint: 15 litres
(C) Blue paint: 10 litres, yellow paint: 30 litres (D) Blue paint: 13 litres, yellow paint: 27 litres

Explanatory Answers

Q1. (D) Let x stand for the percentage who own both a car and a motorcycle. Then
(The %age who own a motorcycle) + (The %age who own a car) - (The %age who own one or the other or both) = 100% own one or other or both.

$$\therefore 15\% + 90\% - A = 100\%$$

$$\Rightarrow 105\% - A = 100\% \Rightarrow A = 5\%$$

The %age of motorcycle owners to who own car is

$$= \frac{5\%}{15\%} = \frac{1}{3} = 33\frac{1}{3}\%$$

Q2. (C) Ratio = 1 : 5 : 4
 Sum of ratio = 1 + 5 + 4 = 10
 Sand = $\frac{5}{10} \times 100 = 50\%$

Q3. (A) Ratio = 5 : 4 : 3
 Sum of ratio = 5 + 4 + 3 = 12
 least share = $\frac{3}{12} \times 24000$
 = Rs. 6000

Q4. (B) Let "x" be the number of parts in 44 hours
 Then 16 : 1280 :: 44 : x
 $\Rightarrow \frac{16}{1280} = \frac{44}{x} \Rightarrow x = \frac{44 \times 1280}{16}$
 $x = \boxed{3520}$

Q5. (C) The ratio of x to y can be written as $\frac{x}{y}$. The ratio of x to y is $\frac{11}{3}$, which can be written as

$$\frac{x}{y} = \frac{11}{3}$$

If $\frac{x}{y} = \frac{11}{3}$, then $2\left(\frac{x}{y}\right) = 2\left(\frac{11}{3}\right)$
 $\frac{2x}{y} = \frac{22}{3}$

Q6. (A) Since 80% of the application were rejected. Therefore, 20% = (100% - 80%) were accepted, the ratio of accepted to rejected is

$$20\% : 80\% = 1 : 4$$

Q7. (D) The ratio of the circumference to the diameter of the circle is π . Therefore,

$$\pi = \frac{C}{d} \Rightarrow \frac{C}{2r} \Rightarrow 2\pi = \frac{C}{r}$$

Q8. (A)

A	B
5 : 2	7 : 3
$= \frac{5}{2} : 1$	$= \frac{7}{3} : 1$
2.5 : 1	= 2.3 : 1

Team A has the better record.

Q9. (D) Clearly, the more workers are there, the less time will be required, therefore; $15 : 40 :: \frac{1}{24} : \frac{1}{x}$

$$\Rightarrow \frac{15}{40} = \frac{x}{24} \Rightarrow x = \frac{15 \times 24}{40} = 9 \text{ days}$$

Q10. (D) It's a direct variation question

$$1280 : 2 :: x : \frac{11}{2}$$

Km h Km 2 h

$$\frac{1280}{2} = \frac{x}{11/2} \Rightarrow 2x = \frac{1280 \times 11}{2}$$

$$\Rightarrow x = 3520 \text{ km}$$

Q11. (D) In this question, if a is 18 and b is 14, then the ratio a : b is 9 : 7 but a + b = 32. The point in

this question that a and b can take on many possible values. It is not possible here to establish one definite value for the sum of a and b.

Q12. (C) $\frac{A \text{ apples}}{n \text{ nickels}} = \frac{A \text{ apples}}{5n \text{ cents}} = \frac{x \text{ apples}}{(10d + 25q)\text{cents}}$

$\Rightarrow \frac{A}{5n} = \frac{x}{10d + 25q} \Rightarrow 5nx = A(10d + 25q)$

$\Rightarrow x = \frac{A5(2d + 5q)}{5n}$

$\Rightarrow x = \frac{A}{n}(2d + 5q)$

Q13. (C) Given ratio 3 : 5 of boys and girls. Total number of students in the class is 24.

Number of boys = $\frac{3}{8} \times 24 = 9$ boys

Number of girls = $\frac{5}{8} \times 24 = 15$ girls

In order to have same number of boys and girls, 6 additional boys would have to enroll.

Q14. (B) This is a direct proportion, because the more sugar, the more flour

$\frac{13}{18} = \frac{10}{x}$
 $13x = 180$

$\Rightarrow x = 13\frac{11}{13}$

Q15. (A)

The ratio 3 : 5 gives (3 + 5) = 8 parts

Blue paints = $\frac{3}{8} \times 40 = 15$ litres

Yellow paints = $\frac{5}{8} \times 40 = 25$ litres

Multiple Choice Questions (MCQs)

Q1. If 15 men can weave 120 meters of cloth in a day, how many meters of cloth can be woven by 35 men in a day?

- (A) 135 m (B) 146 m
 (C) 128 m (D) 168 m

Q2. If two items cost c cents, how many items can be purchased for x cents?

- (A) $\frac{x}{2c}$ (B) $\frac{2c}{x}$
 (C) $\frac{2x}{c}$ (D) $\frac{cx}{2}$

Q3. If four cows produce 4 cans of milk in 4 days, how many days does it take to produce 8 cans of milk?

- (A) 1 (B) 2
 (C) 4 (D) 8

Q4. To ride a ferry, the total cost T is 50 cents for the car and driver and c cents for each additional passenger in the car. What is the total cost for a car with n persons in the automobile?

- (A) $T = n + c$ (B) $T = 50 + nc$
 (C) $T = cn$ (D) $T = 50 + c(n - 1)$

- Q5. Park, Jack and Galvin distributed price money of x dollars among themselves. Park received $\frac{3}{10}$ of what Jack and Galvin together received. Jack received $\frac{3}{11}$ of what Park and Galvin together received. What is the ratio of the amount received by Park to the amount received by Jack?
- (A) 7 : 8 (B) 8 : 7
(C) 10 : 11 (D) 14 : 13
- Q6. If a copier makes 3 copies every 4 seconds, then continuous at this rate, how many minutes will it take to make 9000 copies?
- (A) 60 (B) 100
(C) 120 (D) 200
- Q7. A hat company ships its hats, individually wrapped, in 8-inch by 10-inch by 12-inch boxes. Each hat is valued at \$7.50. If the company's latest order required a truck with at least 288,000 cubic inches of storage space in which to ship the hats in their boxes, what was the minimum value of the order?
- (A) \$960 (B) \$1350
(C) \$2250 (D) \$2050
- Q8. Asim's Taxi Service charges an initial fee of \$ 2.25 at the beginning of a trip and an additional charge of \$ 0.35 for each $\frac{2}{5}$ of a mile traveled. What is the total charge for a trip of 3.6 miles?
- (A) \$ 3.15 (B) \$ 5.40
(C) \$ 4.80 (D) \$ 5.05
- Q9. If Scott has earned x dollars by working 3 days a week at a constant daily rate for w weeks, which of the following represent his daily wage?
- (A) $\frac{x}{3w}$ (B) $\frac{w}{3x}$
(C) $\frac{3w}{x}$ (D) $\frac{xw}{3}$
- Q10. If Finn was 18-month old one year ago, how old was he in months, x months ago?
- (A) $x - 30$ (B) $30 - x$
(C) $x - 12$ (D) $24 - x$

Explanatory Answers

- Q1. (D) Cloth woven by 25 men = 120 m
 Cloth woven by 1 man = $\left(\frac{120}{25}\right)$ m
 Cloth woven by 35 men = $\left(\frac{120}{25} \times 35\right)$ m
 $= \frac{24}{5} \times 35 = 24 \times 7$
 $= 168$ m

Correct answer is choice D.

- Q2. (C) Items purchased for c cents = 2
 Items purchased for one cent = $\frac{2}{c}$
 Items purchased for x cent = $\frac{2}{c} \times x$
 $= \frac{2x}{c}$

Correct answer is choice C.

- Q3. (D) Four cows produce one can of milk in one day. Therefore, eight cows could produce two cans of milk in one day. In four days, eight cows will be able to produce eight cans of milk.

Q4. (D) Since, the driver's fee is paid with the car, the charge for $n - 1$ persons = $c(n - 1)$ cents; cost of car and driver = 50 cents. Therefore, $T = 50 + c(n - 1)$.

Q5. (D) Let the amount received by Park, Jack and Galvin be P, J and G respectively. Since, the prize money, x , was distributed to Park, Jack and Galvin, the amount that Jack and Galvin together received equal $x -$ (the amount received by Park) = $120 - P$.

Since, we are given that Park received $\frac{3}{10}$ of what Jack and Galvin together received, we have the equation.

$$P = \left(\frac{3}{10}\right)(x - P)$$

$$P + \frac{3P}{10} = \frac{3x}{10}$$

$$\frac{13P}{10} = \frac{3x}{10}$$

$$P = \left(\frac{3}{10} \times \frac{10}{13}\right) \times x$$

$$P = \frac{3x}{13}$$

Similarly, we are given that Jack received $\frac{3}{11}$ of what Park and Galvin together received $(x - J)$.

We have the equation

$$J = \left(\frac{3}{11}\right)(x - J)$$

$$\Rightarrow J = \frac{3x}{14}$$

Now $P : J = \frac{3x}{13} : \frac{3x}{14}$
 $= 14 : 13$

Hence, correct answer is choice D.

Q6. (D) At 3 copies every 4 seconds, the copier will finish the batch in $(9000/3) \times 4$ seconds, or 12000 seconds. There are 60 seconds in a minute. So,

$$\frac{12000}{60} = 200 \text{ minutes}$$

Q7. (C) An $8 \times 10 \times 12$ -inch box contains 960 cubic inches-288000 total cubic inches divided by 960 cubic inches per box equals 300 boxes. 300 boxes times \$7.50 per hat equals \$ 2250.

Q8. (B) 3.6 miles divided by $2/5$ equals 9, so the total charge is \$ 2.25 + $(9 \times \$ 0.35) = \$ 5.40$.

Q9. (A) His daily wage can be determined by dividing his total income by the total number of days he has worked. x is his income, and $3w$ is the total number of days he has worked, so $\frac{x}{3w}$ is his daily wage.

Q10.(B) If Finn was 18-month old 1 year ago, then he is now $18 + 12 = 30$ months old, $30 - x$ represent his age x months ago.

Average

In Mathematics, average is a representative of a number of given quantities. Average is of several kinds.

METHOD OF FINDING AVERAGE

To find average of any number of quantities of the same kind is to add all the items together and then divide the sum by the number of items.

$$\therefore \text{Average} = \frac{\text{Sum of all the items}}{\text{No. of items}}$$

Model Examples

Example : The average daily temperature from 9th January to 16th January (both inclusive) was 38.6° and that from the 10th to 17th January (inclusive) was 39.2°. What was the temperature on 17th January?

Solution: Total temp. from 9th Jan. to 16th Jan.

$$= 38.6 \times 8^{\circ}\text{C}$$

$$= 308.8^{\circ}\text{C}$$

Since the temp. on 9th = 34.6°C

$$\therefore \text{Total temp. from 10th Jan. to 16 Jan.}$$

$$= 308.8 - 34.6$$

$$= 274.2^{\circ}\text{C}$$

Total temp. from 10 to 17th Jan.

$$= 39.2 \times 8^{\circ}\text{C}$$

$$= 313.6^{\circ}\text{C}$$

$$\therefore \text{Temp on 17th Jan.} = 313.6 - 274.2$$

$$= 39.4^{\circ}\text{C}$$

Multiple Choice Questions (MCQs)

Q1. The average of even integers from 2 to 100 inclusive is:

- (A) 49 (B) 52
(C) 51 (D) 50

Q2. What is the average of first hundred natural numbers?

- (A) 50 (B) 50.5
(C) 49.5 (D) 100

Q3. What is the average of x, y and z ? If $x + y = 5, y + z = 8$ and $x + z = 11$.

- (A) $\frac{11}{3}$ (B) $\frac{1}{2}$
(C) $\frac{13}{5}$ (D) 4

Q4. The average of five numbers is 54. If three of the numbers are 26, 28 and 30, what is the average of the other two?

- (A) 91 (B) 93
(C) 54 (D) 186

Q5. Which of the following is the average of $x^2 - 16, 39 - x^2$ and $3x + 10$?

- (A) $x + 3$ (B) $2x + 13$

- (C) $x + 11$ (D) $\frac{x + 11}{3}$

Q6. 8 students in a class obtained 60%, 3 obtained 75%, 2 obtained 80% and 7 obtained 45% in a class test. What is the average marks?

- (A) 49% (B) 59%
(C) 29% (D) 51%

Q7. The average number of goals a team has scored in 7 matches is 8. They averaged 10 goals for the first 3 matches and they scored 5 goals in each of the next two matches. What is the average score of the last two matches?

- (A) 5 goals (B) 4 goals
(C) 6 goals (D) 8 goals

Q8. If the mean (average) of 6 numbers is 4.5. What is the sum of the numbers?

- (A) 0.75 (B) 10.5
(C) 12 (D) 27

Q9. A worker is paid R rupees per hour for the first 8 hours daily. For every hour

after the first 8 hours, she is paid S rupees per hour. If she works 12 hours in one day, what is her average hourly for the day?

- (A) $8R + S$ (B) $\frac{8R + 4S}{4}$
 (C) $\frac{12R - 8S}{4}$ (D) $\frac{2R + S}{3}$

Q10. Asim had an average of 60 on his first four math tests. After taking the next test, his average dropped to 58. Find his recent test grade.

- (A) 40 (B) 50
 (C) 48 (D) 32

Q11. If $a + b = 8$, $b + c = 9$, and $c + a = 11$, what is the average of a , b and c ?

- (A) $\frac{14}{3}$ (B) $\frac{28}{3}$

- (C) $\frac{14}{6}$ (D) $\frac{7}{3}$

Q12. If the average of 3, 5, 10 and S is 6, what is the value of S ?

- (A) 4 (B) 6
 (C) 12 (D) 0

Q13. What is the average of 3^{10} , 3^{20} and 3^{30} ?

- (A) 3^{59} (B) $3^9 + 3^{19} + 3^{29}$
 (C) 3^{57} (D) $3^{11} + 3^{21} + 3^{31}$

Q14. If $20x + 20y = 70$, what is the average of x and y ?

- (A) $\frac{7}{2}$ (B) 7

- (C) $\frac{7}{4}$ (D) $\frac{4}{7}$

Q15. Which of the following is the average of $x^4 - 20$, $40 - x^4$, and $3x + 4$?

- (A) $x^4 - 24$ (B) $x + 8$
 (C) $x^4 + 3x + 24$ (D) $x + 24$

Explanatory Answers

Q1. (C) As sum of the first n even numbers $= n(n+1)$
 Now, the sum of even numbers from 2 to 100 is
 $2 + 4 + 6 + 8 + \dots + 100$ (or 50 even number)

$$= 50(50 + 1) = 2550$$

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

$$= \frac{2550}{50} = 51$$

Q2. (B) The first 100 natural numbers are $\{1, 2, 3, \dots, 100\}$

Now, sum of all the first n numbers $= \frac{n(n+1)}{2}$ ✓

$$\begin{aligned} \text{Sum of first 100 natural numbers} &= \frac{100(100+1)}{2} \\ &= 5050 \end{aligned}$$

$$\text{Now, average} = \frac{\text{Sum of numbers}}{\text{Number of terms}}$$

$$= \frac{5050}{100} = 50.5$$

Short-cut: The average of first " n " natural number is $\frac{n+1}{2}$.

$$\text{Thus, average} = \frac{100+1}{2} = \frac{101}{2} = 50.5$$

Q3. (D) Adding the given three equations:

$$(x + y) + (y + z) + (z + x) = 5 + 8 + 11$$

$$2x + 2y + 2z = 24$$

$$2(x + y + z) = 24$$

Dividing both sides by 2

$$x + y + z = 12$$

Now average of x, y and z is

$$\frac{x + y + z}{3} = \frac{12}{3} = 4$$

Q4. (B) Let the missing numbers be a and b , then by given condition,

$$\frac{a + b + 26 + 28 + 30}{5} = 54$$

$$a + b + 84 = 270 \text{ (Multiplying both sides by 5)}$$

$$a + b = 186$$

Hence average of a and b is

$$\frac{a + b}{2} = \frac{186}{2} = 93$$

Q5. (C) Average = $\frac{\text{Sum of the terms}}{\text{No. of terms}}$

$$= \frac{x^2 - 16 + 39 - x^2 + 3x + 10}{3}$$

$$= x + 11$$

Q6. (B) 8 students with 60%, total = 480 marks

3 students with 75%, total = 225 marks

2 students with 80%, total = 160 marks

7 students with 45%, total = 315 marks

∴ 20 students obtain a total = 1180 marks

$$\therefore \text{Average} = \frac{1180}{20} = 59\%$$

Q7. (D) Total goals for 7 matches = $7 \times 8 = 56$

Total goals for 3 matches with average score of 10 = 30

Total goals for 2 matches with average score of 5 = 10

∴ Total goals for remaining 2 matches = $56 - 30 - 10 = 16$

Average goals in last two matches = $\frac{16}{2}$

$$= 8 \text{ goals}$$

Q8. (D) Average of 6 numbers = $\frac{\text{Sum of numbers}}{6}$

$$\Rightarrow \text{Sum of the numbers} = (\text{Average of 6 numbers}) \times 6 = 4.5 \times 6 = 27$$

Q9. (D)

For first 8 hours, she is paid = $8R$

Next 4 hours, she is paid = $(12 - 8) = 4S$

Total pay = $8R + 4S$

$$\text{Average} = \frac{8R + 4S}{12} = \frac{2R + S}{3}$$

Q10. (B) Let " x " be the required grade, then

$$\frac{4(60) + x}{5} = 58$$

$$\Rightarrow 240 + x = 290 \Rightarrow x = 290 - 240 = 50$$

Q11. (A)

$$\frac{(a+b) + (b+c) + (c+a)}{3} = \frac{8+9+11}{3}$$

$$\Rightarrow \frac{2(a+b+c)}{3} = \frac{28}{3}$$

$$\Rightarrow a+b+c = 14 \quad \dots\dots\dots(i)$$

Put $a+b = 8 \Rightarrow 8+c = 14 \Rightarrow \boxed{c=6}$

Now put $b+c = 9 \Rightarrow a+9 = 14 \Rightarrow \boxed{a=5}$

again put $c+a = 11 \Rightarrow 11+b = 14 \Rightarrow \boxed{b=3}$

Average of a, b and c = $\frac{6+5+3}{3} = \boxed{\frac{14}{3}}$

Q12. (B) $\frac{3+5+10+S}{4} = 6 \Rightarrow 18+S = 24 \Rightarrow \boxed{S=6}$

Q13. (B) $\frac{3^{10} + 3^{20} + 3^{30}}{3} = (3^{10} + 3^{20} + 3^{30})3^{-1}$

$$= 3^{10-1} + 3^{20-1} + 3^{30-1}$$

$$= 3^9 + 3^{19} + 3^{29}$$

Q14. (C) $20x + 20y = 70 \Rightarrow 20(x+y) = 70 \Rightarrow x+y = \frac{7}{2}$

\Rightarrow Average of x and y = $\frac{x+y}{2} = \frac{\frac{7}{2}}{2 \times 2} = \boxed{\frac{7}{4}}$

Q15. (B) $\frac{(x^4 - 20) + (40 - x^4) + (3x + 4)}{3} = \frac{3x + 24}{3} = \frac{3(x+8)}{3}$

$$= x + 8$$

Sequences & Series

Sequence: A sequence is an ordered list of numbers. The following is a sequence of even numbers:

$$2, 4, 6, 8, \dots$$

Term of a Sequence: A term of a sequence is identified by its position in the sequence. In the following sequence:

$$1, 3, 5, 7, \dots$$

1 is the first term, 3 is the second term, etc. The ellipses symbol (...) indicates that the sequence continues forever.

Arithmetic Progressions: An arithmetic progression is a sequence in which the difference between any two consecutive terms is the same. This is the same as saying; each term exceeds the previous term by a fixed amount. For example,

$$0, 4, 8, 12, \dots$$

is an arithmetic progression in which the common difference is 4.

The following sequence

$$-7, 0, 7, 14, 21, \dots$$

is arithmetic with a common difference of 7.

Finding the Sum of Arithmetic Sequence:

Since, each term of an arithmetic progression exceeds the previous term by a fixed amount. Therefore, we get the following:

- First Term $a + 0d$
- Second Term $a + 1d$
- Third Term $a + 2d$
- Fourth Term $a + 3d$
-
-
- n th Term $a + (n - 1)d$

In above terms, a is the first term and d is the common difference. The formula generates the n th term. The sum of the first n terms of an arithmetic progression is:

$$S_n = \frac{n}{2} \{2a + (n-1)d\}$$

Example 1:

Find the next term in the series:

- 3, 9, 19, 33, 51, ...

Solution:

Write out the series of increments: 6, 10, 14, 18... (each term is the difference between two terms of the original series? This series is an A.P. whose next term is 22. Adding 22 to the term 51 from the original series produce the next term, 73.

Multiple Choice Questions (MCQs)

- Q1. A sequence of numbers $a_1, a_2, a_3, \dots, a_n$ is generated by the rule $a_{n+1} = 2a_n$. If $a_7 - a_6 = 96$, then what is the value of a_7 ?
- (A) 98 (B) 198
(C) 192 (D) 92
- Q2. ✓ The 9th term and common difference of an A.P. are -6 and $\frac{5}{4}$ respectively. The 25th term is:
- (A) 21 (B) -18
(C) 14 (D) -21
- Q3. The ratio of the 7th to the 3rd term of an A.P. is $12 : 5$. Find the ratio of 13th to the 4th term:
- (A) $\frac{3}{5}$ (B) $\frac{13}{10}$
(C) $\frac{10}{3}$ (D) $\frac{7}{10}$
- Q4. The n th term of the sequence $a_1, a_2, a_3, \dots, a_n$ is defined as $a_n = (a_{n-1})$. The first term $a_1 = -1$. What is the value of a_5 ?
- (A) -1 (B) 1
- Q5. If the 12th term of an A.P. is -13 and the sum of the first four terms is 24. Then, the sum of first 10 terms is:
- (A) 0 (B) 1
(C) 5 (D) -5
- Q6. The sum of the first n terms of a series is 31, and the sum of the first $n - 1$ terms of the series is 20. What is the value of n th term in the series?
- (A) 8 (B) 10
(C) 18 (D) 19
- Q7. In the sequence a_n , the n th term is defined as $(a_{n-1} - 1)^2$. If $a_1 = 4$, then what is the value of a_2 ?
- (A) 7 (B) 5
(C) 6 (D) 9
- Q8. The sequence of numbers a, ar, ar^2 and ar^3 are in Geometric Progression. The sum of the first four terms in the series is 5 times the sum of first two terms and $r \neq -1$. How many times larger is the fourth term than the second term?
- (A) 4 (B) 6

(C) 5

(D) 8

Q9. The common ratio of a G.P. is $-\frac{4}{5}$ and the sum to infinity is $\frac{80}{9}$. Find the first term:

(A) 12

(B) 16

(C) 20

(D) 24

Q10. In a Geometric Progression, the first term is 7, the last term 448 and the sum 889. Find the common ratio:

(A) 6

(B) 4

(C) 3

(D) 2

Q11. A series has three numbers a , ar , and ar^2 . In the series, the first term is twice the second term. What is the ratio of the sum of the first two terms to the sum of the last two terms in the series?

(A) 1 : 2

(B) 3 : 1

(C) 1 : 4

(D) 2 : 1

Explanatory Answers

Q1.(C) Substituting $n = 6$ in the given rule $a_{n+1} = 2a_n$

$$a_{6+1} = 2a_6$$

or

$$a_7 = 2a_6$$

Also, given that

$$a_7 - a_6 = 96$$

$$a_7 - \frac{a_7}{2} = 96$$

$$\frac{a_7}{2} = 96$$

$$\Rightarrow a_7 = 192$$

The correct answer is choice C.

Q2.(C) If a be the first term and d be the common difference of an A.P. Then

$$a_n = a + (n - 1)d$$

$$a_9 = a + (9 - 1)d$$

$$a_9 = a + 8d$$

$$a_9 = a + 8 \times \frac{5}{4} \Rightarrow a_n = a + 10$$

$$-6 = a + 10 \Rightarrow a = -6 - 10$$

or

$$\boxed{a = -16}$$

Now

$$a_{25} = a + (25 - 1)d$$

$$= -16 + 24 \times \frac{5}{4}$$

$$= -16 + 30$$

$$\boxed{a_{25} = 14}$$

Correct answer is choice C.

Q3.(C) Let a be the first term and d the common difference of the A.P. Then

$$\frac{a + 6d}{a + 2d} = \frac{12}{5}$$

$$\Rightarrow 5(a + 6d) = 12(a + 2d)$$

$$\Rightarrow 5a + 30d = 12a + 24d \Rightarrow 5a - 12a + 30d - 24d = 0$$

$$\Rightarrow -7a + 6d = 0$$

$$\Rightarrow \boxed{a = \frac{6}{7}d}$$

Now,
$$\frac{13\text{th term}}{4\text{th term}} = \frac{a+12d}{a+3d} = \frac{\frac{6}{7}d+12d}{\frac{6}{7}d+3d}$$

$$= \frac{90}{27} = \frac{10}{3}$$

Correct answer is choice C.

Q4.(A) The rule of the given sequence is

$$a_n = -(a_{n-1})$$

Putting $n = 2$ and 3 in the given sequence, we have

$$a_2 = -(a_{2-1}) \Rightarrow a_2 = -a_1$$

$$\Rightarrow a_2 = -(-1) \Rightarrow a_2 = 1$$

Now $a_3 = -(a_{3-1}) \Rightarrow a_3 = -a_2$

$$\Rightarrow a_3 = -(1) \Rightarrow \boxed{a_3 = -1}$$

Similarly, we get that each even numbered term equals 1 and each odd numbered term equals -1. Since a_5 is an odd numbered term, it equals -1. The correct answer is choice A.

Q5.(A) Let a be the first term and d be the common difference of the A.P.

Then n th term $= a + (n - 1)d$

$$\therefore a_{12} = a + (12 - 1)d \Rightarrow a_{12} = a + 11d$$

$$\Rightarrow -13 = a + 11d \quad \dots(i)$$

Now, $S_n = \frac{n}{2} \{2a + (n-1)d\}$

$$S_4 = \frac{4}{2} \{2a + (4-1)d\}$$

$$S_n = 2(2a + 3d)$$

$$\Rightarrow 24 = 2(2a + 3d)$$

$$\Rightarrow 2a + 3d = 12 \quad \dots(ii)$$

Multiplying equation (i) by 2 and subtracting from (ii), we have

$$2a + 3d = 12$$

$$2a + 22d = -26$$

$$\begin{array}{r} - \quad - \quad + \\ \hline \end{array}$$

$$-19d = 38$$

$$\Rightarrow \boxed{d = -2}$$

Substituting the value of -2 in (ii), we have

$$2a + 3(-2) = 12$$

$$2a - 6 = 12$$

$$\Rightarrow 2a = 18$$

$$\Rightarrow \boxed{a = 9}$$

Now

$$S_{10} = \frac{n}{2} \{2a + (10-1)d\}$$

$$S_{10} = \frac{10}{2} \{2(9) + 9(-2)\}$$

$$S_{10} = 5(18 - 18) \Rightarrow \boxed{S_n = 0}$$

Correct answer is choice A.

Q6.(B) (The sum of the first n terms of a series) = (The sum of the first $(n - 1)$ term) + (The n th term)
Substituting the given values in the equation gives

$$31 = 21 + n\text{th term}$$

$$\Rightarrow n\text{th term} = 31 - 21$$

$$\Rightarrow n\text{th term} = 10$$

Correct answer is choice B.

Q7.(D) Given that

$$a_n = (a_{n-1} - 1)^2$$

Replacing n by 2, we have

$$a_2 = (a_{2-1} - 1)^2$$

$$a_2 = (a_1 - 1)^2 \quad \dots(i)$$

Given that $a_1 = 4$
 Putting the value of a_1 in (1), we have

$$a_2 = (4 - 1)^2$$

$$\boxed{a_2 = 9}$$

Correct answer is choice D.

Q8.(A) In the given progression, the sum of the first two terms is $a + ar$, and the sum of the first four terms is $a + ar + ar^2 + ar^3$. Since "the sum of the first four terms in the series is 5 times the sum of the first two terms." Thus,

$$a + ar + ar^2 + ar^3 = 5(a + ar)$$

Divide both sides by $(a + ar)$

$$\frac{a + ar + ar^2 + ar^3}{(a + ar)} = \frac{5(a + ar)}{(a + ar)}$$

$$\frac{(a + ar) + r^2(a + ar)}{a + ar} = 5$$

$$\frac{(a + ar)(1 + r^2)}{a + ar} = 5$$

$$1 + r^2 = 5$$

$$\Rightarrow r^2 = 4$$

Now, the fourth term is $ar^3/ar = 4$ times the second term.
 Hence, correct answer is choice A.

Q9.(B) The sum to infinity, $S_\infty = \frac{a}{1-r}$

$$\therefore \frac{80}{9} = \frac{a}{1 - \left(\frac{-4}{5}\right)}$$

$$= \frac{a}{5+4}$$

$$= \frac{a}{5}$$

$$\Rightarrow \frac{80}{9} = \frac{a}{5} \Rightarrow \frac{80}{9} = a \times \frac{5}{9}$$

$$\Rightarrow 80 = 5a \Rightarrow \boxed{a = 16}$$

Hence, the first term of the geometric progression is 16.

Q10.(D) Here, $a = 7, \ell = a_n = 448, S_n = 889$

Let r be the common ratio

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{a-lr}{1-r} \Rightarrow 889 = \frac{7-448r}{1-r}$$

$$\Rightarrow 889(1-r) = 7-448r$$

$$\Rightarrow 889 - 889r = 7 - 448r$$

$$\Rightarrow 889 - 7 = 889r - 448r$$

$$\Rightarrow 882 = 441r$$

$$\Rightarrow r = \frac{882}{441}$$

$$\Rightarrow r = 2$$

Hence, correct answer is choice D.

Q11.(D) Since, the first term in the series is twice the second term, we have $a = 2(ar)$.

$$\Rightarrow 1 = 2r \Rightarrow \boxed{r = \frac{1}{2}}$$

The three numbers a, ar, ar^2 becomes $a, a(1/2),$ and $a(1/2)^2$ or $a, \frac{a}{2}, \frac{a}{4}$.

Now, The sum of the first two terms $= a + \frac{a}{2}$

The sum of the last two terms $= \frac{a}{2} + \frac{a}{4}$

Setting ratio

$$\frac{a + \frac{a}{2}}{\frac{a}{2} + \frac{a}{4}}$$

$$= \frac{\frac{2a+a}{2}}{\frac{2a+a}{4}} = \frac{\frac{3a}{2}}{\frac{3a}{4}}$$

$$= \frac{3a}{2} \times \frac{4}{3a}$$

$$= \frac{2}{1} \quad \text{or} \quad 2:1$$

Hence, correct answer is choice D.

Ushr

To use "Ushri" land for agriculture and getting benefit therefrom will make Zakaat compulsory on the produce. This Zakaat on agricultural produce is called "Ushr" (tithe) i.e. one-tenth of the produce is due in most of cases and in some cases half of it (one-twentieth) is charged.

There are many conditions and circumstances which make land Ushri. For example, the land would be entitled to be called Ushri, if Muslims conquered a country and distributed lands among Muslim soldiers or the subjects of that country surrendered and embraced Islam at their own without any resistance or battle or the said land is irrigated by Ushri water. Most of the lands of the Muslims in Indo-Pak fall within the category of Ushr i.e. either full Ushr or half Ushr is due on them.

If a family land is irrigated by canal or channel water, Ushr i.e. one-tenth of the produce is due thereon and if irrigated by shoal or bucket then half Ushr (one-twentieth) is due. Similarly, half Ushr is also due on land, field irrigated by the water bought from somebody who is owner of it.

Ushr is due on every kind of cereals like wheat, barley, millet, paddy and dry fruits like walnut, almond and vegetables like melon, water-melon, cucumber, eggplant irrespective of less or more cultivation.

Ushr or half Ushr, as the case may be, will be due on total produce of agricultural commodities on which Ushr or half Ushr is due without deducting the expenses of cultivation like peasants' labour, ploughing charges and price of seeds etc. Moreover, rainy, river, spring or well water is called Ushri water. Ushr will be due on the produce cultivated by such water. Ushr will be collected from only Muslims. Contrarily, if a Muslim buys "Kharaaji" a piece of land from a "Zimmi" (a loyal non-Muslim citizen of Islamic state), the land will remain "Kharaaji". Ushr of this land will not be charged from the Muslim buyer but Kharaaj will be charged instead.

There are many conditions which render a land Kharaaji. For instance, if a Muslim army conquered an enemy state and later returned the land to them whether to oblige them or gave it to other non-Muslims or the subjects of that state surrendered negotiating a peace agreement with the Muslim army but did not embrace Islam or a "Zimmi" bought Ushri land from a Muslim or a Muslim bought Kharaaji land or a land is irrigated by Kharaaji water, in all such cases the land is called Kharaaji land. The water of the canal dug by non-Muslims before the arrival of Muslims is Kharaaji water or un-believers dug a well but now it is under the control of Muslims or was dug in Kharaaji land, is also Kharaaji. The produce of the land irrigated by such water will be subjected to a certain amount of Kharaaj (not Ushr). It may be half, one-third, one-fourth of the produce or any other quantity thereof may be made compulsory.

Completion of full one year is no condition for Ushr's being due. If cultivation is done on the land more than once in a year Ushr will be due on each of cultivation. Nisaab is no condition for Ushr's being due. If agricultural produce, irrespective of perishable or imperishable, is even one kg. Ushr will become due on it. There is also no condition that the cultivator should be owner of the land. If it is an unclaimed land even then Ushr is due on its produce if cultivated.

Ushr is due on agricultural produce (not on person). If one dies before paying Ushr and his crop is still there, Ushr will be charged on the produce. If crop produce is destroyed completely by flood, torrential rain, fire, hot wind or eaten up by locusts, Ushr and Kharaaj will stand annulled but if some remains untouched, Ushr will be charged on the intact produce.

And in case, the crop produce is eaten away by animals, Ushr will not be annulled but will remain due. Similarly, if it is destroyed before harvest, Ushr will not be charged otherwise will be due.

If crop is sold before it is ready for harvest then the buyer should pay Ushr and if it is ripe at the time of sale then the seller should pay Ushr. In case, both the land and crop are sold or only land is sold and there is also enough time to cultivate the land once more before the completion of one year then Ushr is due on the buyer otherwise on the seller.

The uses of Ushr and half-Ushr are the same as that of Zakaat which are to follow this lesson. As for Kharaaj, its income is used for the greater good of Muslim community at large. For example, construction of mosques, and other related expenses like stipend of Imam and Mu'azzin, salaries of teachers of religious seminaries, scholarship to students seeking Islamic education, knowledge and religious scholars who are serving Islam by doing research work, imparting Islamic knowledge, issuing religious edicts, decrees etc. It can also be used for building bridges, inns etc.

Zakat

'Zakat' is paid by a person who is Sahib-e-Nisab under Islamic principles @ 2.50% per annum on his saving in monetary terms, gold and silver and tradeable goods according to the set principles of Islam.

Q.1. What is the amount of Zakat payable by a person who saves Rs. 13500?

Sol.

$$\begin{aligned} \text{Amount of Zakat @ } 2\frac{1}{2}\% &= \frac{5}{200} \times 13500 \\ &= \text{Rs. } 337.5 \end{aligned}$$

Q.2. What is the amount saved if Rs. 187.50 is paid as Zakat?

Sol.

$$\text{Zakat payable} = 2\frac{1}{2}\% \text{ of one's savings}$$

$$\therefore \text{ Zakat paid on savings} = \text{Rs. } 187.50$$

$$\begin{aligned} \therefore \text{ Saving} &= \text{Rs. } 187.50 + \frac{5}{200} \\ &= \text{Rs. } 187.50 \times \frac{200}{5} \\ &= \text{Rs. } 7500.00 \end{aligned}$$

Q.3. At a rate of $2\frac{1}{2}\%$ p.a. (per annum) how much Zakat will be paid on a wealth of Rs. 150850?

Sol.

$$\text{Wealth} = \text{Rs. } 150850$$

$$\text{Rate of Zakat} = 2\frac{1}{2}\%$$

$$\text{Amount of Zakat payable} = \frac{5}{2} \times \frac{150850}{100} = \text{Rs. } 3771.25$$

Q.4. Find the amount of Zakat paid by Zakir at a rate of $2\frac{1}{2}\%$ p.a. on his wealth value at Rs. 89,000.

Sol.

$$\text{Wealth} = \text{Rs. } 89,000.00$$

$$\text{Rate of Zakat} = 2\frac{1}{2}\%$$

$$\text{Amount of Zakat payable} = \frac{5}{2} \times \frac{89000}{100} = \text{Rs. } 2225$$

Q.5. Mukhtar's wife had a jewellery valued at Rs. 295000. Find the amount of Zakat payable at a rate of $2\frac{1}{2}\%$ p.a.?

Sol.

$$\text{Wealth} = \text{Rs. } 295000$$

$$\text{Rate of Zakat} = 2\frac{1}{2}\%$$

$$\text{Amount of Zakat payable} = \frac{5}{2} \times \frac{295000}{100} = \text{Rs. } 7375$$

Q.6. A man paid Zakat of Rs. 312.50 at the rate of $2\frac{1}{2}\%$ of his wealth. What is the value of his wealth?

Sol.

$$\text{Zakat paid} = \text{Rs. } 312.50$$

$$\text{Rate of Zakat} = 2\frac{1}{2}\%$$

$$\text{Value of wealth} = ?$$

$$2\frac{1}{2}\% \text{ of wealth} = \text{Rs. } 312.50$$

$$\text{or } \frac{5}{200} \times \text{wealth} = \text{Rs. } 312.50$$

$$\text{or wealth} = \text{Rs. } \frac{312.50 \times 200}{5}$$

$$\text{Required value of wealth} = \text{Rs. } 12,500$$

Q.7. Ijaz paid Zakat of Rs. 3705 at $2\frac{1}{2}\%$ p.a. Find the value of his wealth.

Sol.

$$\text{Wealth} = \text{Rs. } 3705$$

$$\text{Rate of Zakat} = 2\frac{1}{2}\%$$

$$\text{Value of wealth} = ?$$

$$\frac{5}{2}\% \text{ of value} = 3705 \text{ or } \frac{5}{2} \times \frac{1}{100} \text{ of wealth} = \text{Rs. } 3705$$

$$\text{or wealth} = 3705 \times \frac{200}{5} = \text{Rs. } 148200$$

Q.8. A man has to pay Zakat on a wealth of Rs. 7,500 at the rate of $2\frac{1}{2}\%$. Find how much Zakat will he have to pay?

Sol.

$$\text{Wealth} = \text{Rs. } 7,500$$

$$\text{Rate of Zakat} = 2\frac{1}{2}\%$$

$$\text{Amount of Zakat} = 2\frac{1}{2}\% \text{ of Rs. } 7,500$$

$$= \frac{5}{2} \times \frac{7500}{100}$$

$$= \text{Rs. } 187.50$$

Word Problems

Multiple Choice Questions (MCQs)

- Q1.** If 5 is subtracted from a certain number, the result is 7 less than twice the number. What is the number?
 (A) 2 (B) $\frac{1}{2}$
 (C) 5 (D) 6
- Q2.** Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?
 (A) 11 (B) 12
 (C) 15 (D) 13
- Q3.** Two-fifth of a certain number is 30. What is the number?
 (A) 75 (B) 25
 (C) 90 (D) 150
- Q4.** Saira weights 25 pounds more than Umbar. If together they weigh 205 pounds, what is the weight of Saira?
 (A) 90 (B) 105
 (C) 115 (D) 135
- Q5.** If the sum of two numbers is 36, and the larger is three times as larger as the smaller, what is the larger number?
 (A) 27 (B) 30
 (C) 15 (D) 18
- Q6.** The sum of integers p and q is 352. The units digits of p is 0. If p is divided by 10, the result is equal to q , what is the value of p ?
 (A) 30 (B) 230
 (C) 320 (D) 32
- Q7.** A soap factory has 30 packers. Each packer can load $\frac{1}{8}$ of a box in 9 minutes. How many boxes can be loaded in $1\frac{1}{2}$ hours by all 20 packers?
 (A) 28 (B) $37\frac{1}{2}$
 (C) 35 (D) $35\frac{1}{2}$
- Q8.** Uzma is 15 years old. Asma is one-third older than Uzma. How many years ago when Asma was twice as old as Uzma is?
- (A) 5 (B) 12
 (C) 15 (D) 10
- Q9.** Moeed is now three times Mohsin's age. Four years from now, Moeed will be y years old. In terms of y , how old will Mohsin be?
 (A) $\frac{x-4}{3}$ (B) $\frac{x+4}{3}$
 (C) $x+4$ (D) $x-4$
- Q10.** If the sum of one-third of a number and twice the same number is 28, the number is:
 (A) 10 (B) 12
 (C) 28 (D) 14
- Q11.** A man's present age is x years. If his age in 8 years will be $\frac{4}{5}$ of what it will be in 20 years, then his present age is:
 (A) 45 (B) 25
 (C) 30 (D) 40
- Q12.** When 42 is added to twice a number, the result is 346, the number is:
 (A) 304 (B) 242
 (C) 152 (D) 265
- Q13.** A man was 26 years old when his daughter was born. Now, he is three times as old as his daughter. How many years old is the daughter now?
 (A) 13 years (B) 22 years
 (C) 15 years (D) 12 years
- Q14.** 13 years ago Shabbir's mother was 7 times as old as he was. She is now 48 years old. How many years old is Shabbir now?
 (A) 28 (B) 18
 (C) 38 (D) 20
- Q15.** If 5 years are added to a man's present age and that age is tripled, he will be 84. What is his present age?
 (A) 18 (B) 23
 (C) 32 (D) 54

Explanatory Answers

Q1. (A) Let the required number be x . Then $x - 5 = 2x - 7$
 $\Rightarrow \boxed{x = 2}$. Thus the correct answer is 2.

Q2. (C) Let
 $x =$ first integer
 $x + 2 =$ second integer
 $x + 4 =$ third integer
 $3(x) = 3 + 2(x + 4)$
 $3x = 3 + 2x + 8$
 $x = 11$
 Third integer is $(x + 4) = \boxed{15}$

Q3. (A) Let the number $= x$, then
 $\frac{2}{5}x = 30$
 $\Rightarrow x = \frac{30 \times 5}{2}$
 $\Rightarrow x = 75$

Q4. (C) Let the weight of Saira $= x$
 and Umber's weight $= y$
 $x + 25 = y$
 and $x + y = 205$
 $\Rightarrow x - y = 25$
 $x + y = 205$
 $2x = 230$
 $x = \frac{230}{2} = 115$ pound

Q5. (A) Let the smaller number $= x$
 Then the larger number $= 3x$
 Now $3x + x = 36$
 $4x = 36$
 $x = 9$
 The larger number is $36 - 9 = 27$

Q6. (C) $p + q = 352$ and $\frac{p}{10} = q \Rightarrow p = 10q$
 $10q + q = 352 \Rightarrow 11q = 352 \Rightarrow q = 32$
 Now $p + 32 = 352 \Rightarrow \boxed{p = 320}$

Q7. (B) 30 packers will load $30 \times \frac{1}{8}$ or $\frac{30}{8}$ boxes in 9 minutes. There are 90 minutes in $1\frac{1}{2}$ hours
 the 30 packers will load $10 \times \frac{30}{8}$ or $37\frac{1}{2}$ boxes in $1\frac{1}{2}$ hours.

Q8. (D) Asma is one-third older or $\frac{1}{3} \times 15 = 5$ years older. Let x be the age of Uzma and $x + 5$ Asma's age. When Asma was twice the age of Uzma, $2x = x + 5$ or $x = 5$. Uzma was 5 years old and Asma was $x = 5$ or 10 years old, twice Uzma's age. Since Uzma is 15 years old now

Uzma was 5 years old 10 years ago.
Q9. (A) Assume x for Moeed and y for Mohsin

$$x \text{ is three times } y \Rightarrow x = 3y$$

$$x \text{ in four years } \Rightarrow x = x + 4$$

$$\Rightarrow x = 3y + 4$$

$$\Rightarrow x - 4 = 3y$$

$$\frac{x-4}{3} = y$$

Q10. (B) Let x be the required number, then

$$\frac{1}{3}x + 2x = 28$$

$$\Rightarrow x + 6x = 84$$

$$\Rightarrow 7x = 84$$

$$\Rightarrow \boxed{x = 12}$$

Q11. (D) Present age = x

$$x + 8 = \frac{4}{5}(x + 20)$$

$$5x + 40 = 4x + 80$$

$$5x - 4x = 80 - 40$$

$$\boxed{x = 40}$$

Q12. (C) Let x be the required number, then

$$2x + 42 = 346$$

$$\Rightarrow 2x = 304$$

$$\Rightarrow \boxed{x = 152}$$

Q13. (A) Let x be the age of man and y be the age of his daughter

$$x - 26 = y \quad \dots\dots\dots(1)$$

$$x = 3y \quad \dots\dots\dots(2)$$

Substituting the value of x in (1),

$$3y - 26 = y$$

$$2y = 26 \Rightarrow \boxed{y = 13}$$

Q14. (B) Let x be the age of Shabbir

$$7(x - 13) = 48 - 13$$

$$7(x - 13) = 35$$

$$x - 13 = 5$$

$$\boxed{x = 18}$$

Q15. (B) Let x be the man's present age, then

$$3(x + 5) = 84$$

$$\Rightarrow x + 5 = 28$$

$$\Rightarrow \boxed{x = 23}$$
