Chapter 19

DAWN OF MODERN PHYSICS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Distinguish between inertial and non-inertial frames of references.
- Describe the postulates of special theory of relativity and its results.
- Understand the NAVASTAR navigation system.
- Understand the concept of black body radiation.
- Know Planck's assumptions.
- Know the origin of quantum theory.
- Show an appreciation of the particle nature of electromagnetic radiation.
- Describe the phenomenon of photoelectric effect.
- Explain photoelectric effect in terms of photon energy and work function.
- Explain the function of photocell and describe its uses.
- Describe Compton's effect.
- Explain the phenomena of pair production and pair annihilation.
- Describe de-Broglie's hypothesis of wave nature of particles.
- Describe and interpret qualitatively the evidence provided by electron diffraction for the wave nature of particles.
- Understand the working principle of electron microscope.
- Understand and describe uncertainty principle.

INTRODUCTION

In the beginning of 20^{th} century, many experimental and theoretical problems relating to physics could not be solved on the basis of Newtonian classical physics. For example the behaviour of matter on the atomic level. The phenomenon of black body radiation, the photoelectric effect and constancy of speed of light could not be explained in terms of laws of classical physics. To explain these observations new concepts were needed. The study of these concepts is known as modern physics.

Q.1 What is relative motion?

Ans. **RELATIVE MOTION**

The change in position of a body with respect to another body is called relative motion. If a ball is thrown up, the up direction is only for that particular place. It will be down position for a person on opposite of 1st observer. This concept of direction is relative. Similarly, rest or motion of object is not same for different observers. For example Cabin walls of moving train are at rest with the respect to passenger while, they are in motion with the respect to observer are stationary on earth. All motions are relative to a person or instrument observing it.

Examples

(i) If we place a body on Earth, it remains at rest unless an unbalanced force is applied upon it. This observation shows that Earth may be considered as an inertial frame of reference.

(ii) If a body is placed in a car moving with a uniform velocity with respect to Earth also remains at rest. So that a car is also an inertial frame of reference.

Q.2 Define frame of reference with its type.

Ans. FRAME OF REFERENCE

The space bounded by three mutually perpendicular co-ordinate axes with respect to which observations are made is called frame of reference. For example If an observation is made with respect to the wall of the room, the room is the frame of reference. Similarly, the cabin of a moving train is also a frame of reference. If observation are made with respect to it. The frame of reference may be at rest or in motion. They have two types

(1) Inertial Frame of Reference

Those frame of references which are at rest or moving with uniform velocity i.e., it has no acceleration called **inertial frame of reference**.

Newton's laws of motion are valid in inertial frame of reference. Inertial frame of reference are also called non-accelerated frame of reference.

(2) Non-inertial Frame of Reference

That frame of reference which moves with some acceleration is called non-inertial frame of reference or accelerated frame of reference.

Example

If the moving car is suddenly stopped or accelerated up is a non-inertial frame of reference.

Newton's first law of motion does not valid in the non-inertial frame of reference.

Q.3 State postulates of special theory of relativity. Explain three results of theory.

Ans. SPECIAL THEORY OF RELATIVITY

In classical mechanics, the velocities of moving objects are negligible as compared to velocity of light. However, relativitics mechanics deals with velocities of moving objects comparable to speed of light.

The part of relativistic mechanics which deals with uniform relativistic velocities is called **special theory of relativity**. On the other hand, the part of relativistic mechanics which deals with accelerated relativistic velocities is called the **general theory of relativity**.

Postulates of Special Theory of Relativity

Special theory of relativity is based on the following postulates

(1) The laws of physics are the same in all inertial frame of reference.

(2) The speed of light in free space has the same value for all observers.

Explanation

If the laws of physics is different for different observers in relative motion. The observer could determine from this difference that which of them were stationary in a space and which were moving. But such distinction does not exist so, the 1st postulate show that there is no way to detect absolute uniform motion.

The 2nd postulate states an experimental fact that speed of light in free space is universal constant i.e., $C = 3.0 \times 10^8$ meter per sec. These simple postulates have far-reaching consequences. These include such phenomena as the slowing down of clocks and contraction of lengths in moving reference frames as measured by a stationary observer. Some interesting results of the special theory of relativity can be summarized as follows without going into their mathematical derivations.

Results of Special Theory of Relativity

According to special theory of relativity, following are the results:

(1) Time Dilation (time increase in motion)

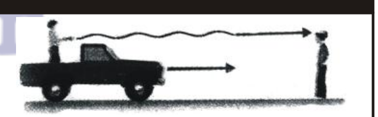
According to special theory of relativity, time is not absolute quantity. It depends upon the motion of frame of reference. Let t_o be the time interval between two events as measured by an observer in stationary frame of reference. This is known as proper time. If the frame of events is moving with uniform relativistic velocity v with respect to observer. The time measured by the observer would not be t_o but it would be t given by

$$t = \frac{t_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

As, $\sqrt{1-\frac{v^2}{c^2}} < 1$

then, $t > t_o$

Do You Know?



The speed of light emitted by flashlight is c measured by two observes, one on the moving

i.e., time has dilated due to relative motion of observer and frame of events. Time dilation is applicable to all timing processes physical, chemical and biological. Even aging process of human body is slowed down by motion at very high speed.

(2) Length Contraction (Length Decreases in Motion)

The distance from earth to star measured by an observer in a moving spaceship would seems smaller than the distance measured by an observer on earth. This effect is known as length contraction. The length of an object or distance between two points measured by an observer who is relatively at rest is called proper length l_o. If the object and an observer are in relative motion with relativistic speed v, then the contracted length is l is given by

$$l = l_o\sqrt{1-\frac{v^2}{c^2}}$$

The length contraction is only along the direction of motion and not at right angle to motion.

(3) Mass Variation

According to special theory of relativity an object whose mass when measured at rest is m_o will have an increased mass m when moving at relativistic velocity v is given by

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

The increase in mass indicates the increase in inertia, so large force is required to accelerate the body.

When $v = c$ then $1-\frac{v^2}{c^2} = 0$ and $m = \propto$

So an infinite force is needed to accelerate the mass since infinite force cannot be applied then a body cannot be accelerated to a speed of light in free space.

Discussion on above Results

In our everyday life, we deal with velocities which are extremely small as compare to velocity of light. Even the earth orbital speed is only 30 km/sec. Whereas velocity of light in free space is 300,000 km/sec. So, under this situation

$$\frac{v}{c} \simeq 0 \quad \text{and hence}$$

$$t = t_o \;,\; l = l_o \;,\; m = m_o$$

No relativistic effects are observe in everyday life and newtanion physics is valid in everyday situation. However, dealing with atomic particles moving with velocities approaching with speed of light, the relativistic effects cannot be neglected. The experimental results are explained by taking Einstein's equations into account.

(4) Mass Energy Relation

According to Einstein special theory of relativity, mass and energy are interconvertable and are related by the relation

$$E = mc^2$$

where m depends upon the speed of object. Energy $E_o = m_o c^2$ correspond to rest mass m_o is called **rest-mass energy**.

As mc^2 is greater than m_oc^2, the difference of energy is due to motion and is called K.E of the mass.

Thus K.E is equal to

$$K.E = mc^2 - m_oc^2$$

$$K.E = (m - m_o)c^2$$

The change in mass is due to change in K.E.

ΔE is given by

$$\Delta m = \frac{\Delta E}{c^2}$$

As c^2 is very large quantity, so sizeable change in mass requires very large change in energy. In our everyday world, energy changes are too small to provide measurable mass changes. However, energy and mass changes in nuclear reactions are found to be exactly in accordance with the above mentioned equations.

NAVSTAR Navigation System

The results of special theory of relativity are put to practical use even in everday life by a modern system of navigation satellites called NAVSTAR. The location and speed anywhere on Earth can now be determined to an accuracy of about 2 cms^{-1}. However, if relativity effects are not taken into account, speed could not be determined any closer than about 20 cms^{-1}. Using these results the location of an aircraft after an hour's flight can be predicted to about 50 m as compared to about 760 m determined by without using relativistic effects.

Q.4 What is black body radiation? Explain intensity distribution diagram.

Ans. BLACK BODY RADIATION

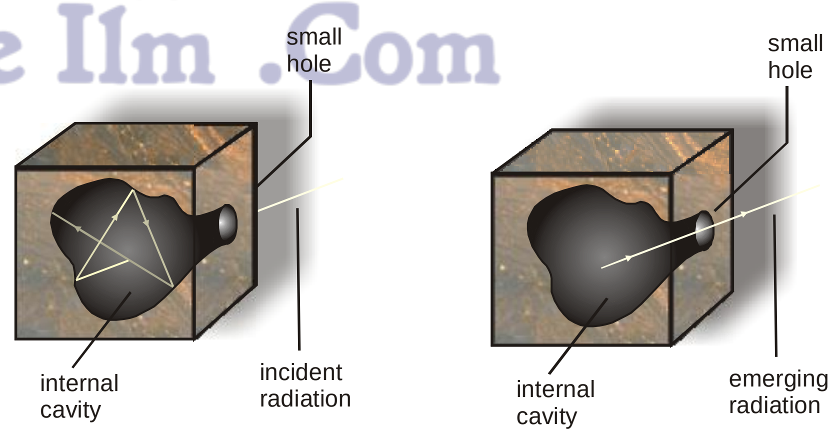
Radiation

The amount of energy emitted from hot body in the form of electromagnetic waves is called radiations or thermal radiations.

When a body is heated, it emits radiations. The radiation emited depend upon the temperature and nature of material of body. However, radiations from black body depend only on the temperature of a black body.

Black Body

It is a cavity in a solid with small opening. The walls of the cavity are made black with suitable black material to make it good absorber. The small opening appears black because the radiations that enters is reflected from the inside walls many times untill completely absorbed i.e., cannot escapes out. Thus black body is a good absorber of radiation. If a black body is heated to higher temperature, the opening of cavity emits radiation like perfect emitter. The radiations from such body are called black body radiations or cavity radiations.



Absorption of radiation   Emission of radiation

Intensity Distribution in Black Body Radiation

Lummer and Pringsheim measured the intensity of radiation emitted from the black body in different wavelengths when the black body was heated through different temperatures. Their experimental arrangement is shown in figure.

The amount of radiation emitted with different wavelength when black body is heated at different temperature have been shown graphically.

The following effects can be studied from the graphs.

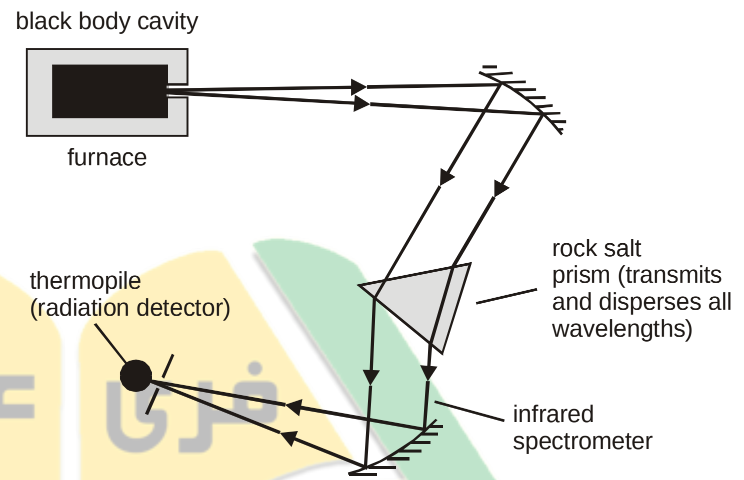
(i) At a given temperature, the energy is not uniformly distributed in black body radiation spectrum.

(ii) **Wein Displacement Law** This law states that; for a particular temperature, the wavelength of maximum intensity (λ_{max}) is inversely proportional to the absolute temperature of the black body that is

$$\lambda_{max} \propto \frac{1}{T}$$

$$\lambda_{max} = \frac{\text{Constant}}{T}$$

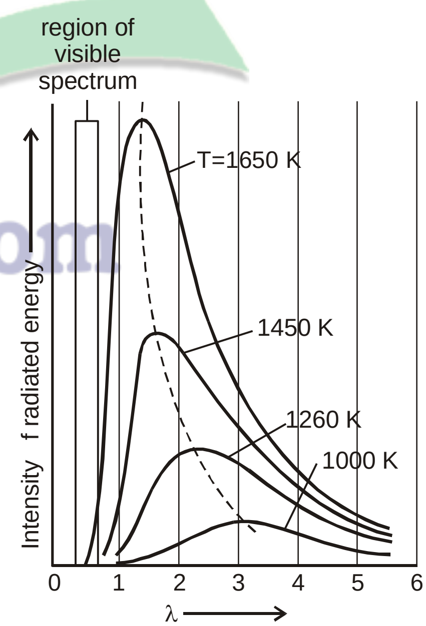
$$\lambda_{max} \times T = \text{Constant}$$



The value of constant is about 2.9×10^{-3} mK is called Wein's constant.

This law tells us that as temperature increases, wavelength λ shifts to shorter values.

(iii) For all wavelengths, an increase in temperature causes an increase in energy emission. The radiation increases with increase in wavelength and at particular wavelength λ_{max}, it has maximum value. With further increase in wavelength, the intensity of radiation decreases.



(iv) **Stephen's Boltzman Law** This law states that area under each curve represents the total energy emitted over all wavelength at a particular temperature and is found to be directly proportional to the fourth power of absolute temperature of black body.

Mathematically

$$E \propto T^4$$

$$E = \sigma T^4$$

Where σ is constant called Stephen's constant and its value is 5.67×10^{-8} watt m^2K^{-4}.

Planck's Assumptions

Electromagnetic theory of radiation could not explain the above energy distribution curves successfully. Maxwell Planck proposed quantum theory of radiation to explain energy distribution curves. His theory is based on following assumptions

(i) The energy is radiated or absorbed by a black body is in the form of packets of energy rather than in the shape of continuous wave. The packets of energy are called quanta. Each quantum has an energy given by

$$E = hf$$

where h is constant and its value is 6.63×10^{-34} J sec. and f is the frequency of radiation.

(ii) The atoms of the cavity behave as harmonic oscillator each having a particular frequency. The energy is emitted or absorbed by the harmonic oscillator shifts from one the quantised state to another of quantised state. The energy emitted is according to relation

$$E = nhf \quad \text{where} \quad n = 1, 2, 3, \ldots\ldots$$

Max Planck proposed this theory in 1900 and got noble prize for it in 1918.

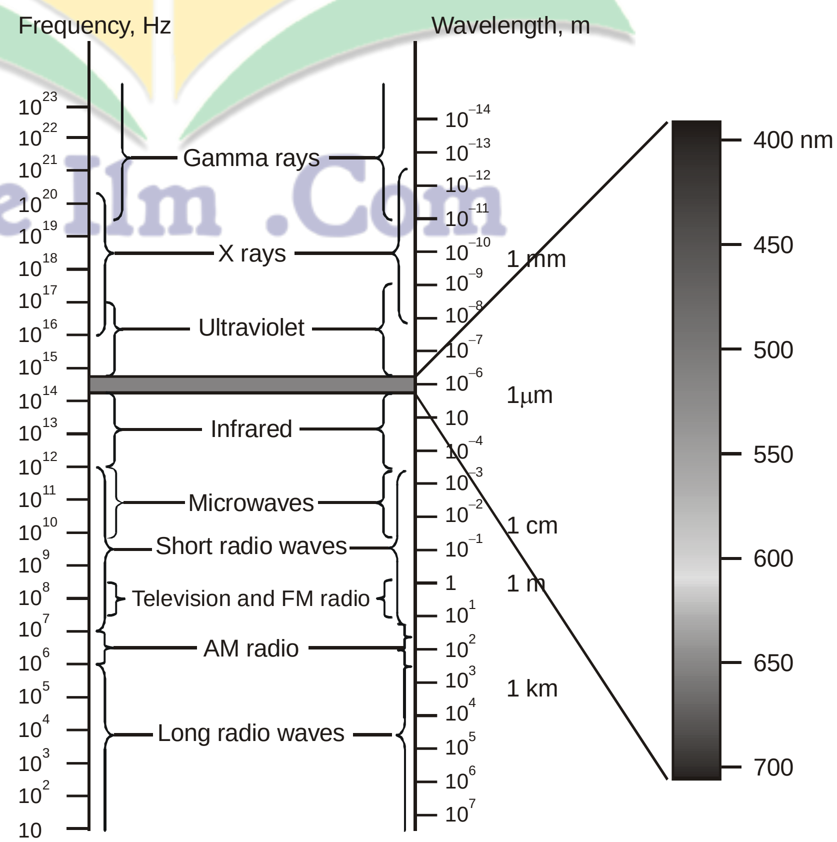
The Photon

The Planck suggested that as matter is not continuous but consist of large number of tiny particles so is the radiation energy from a source. He assumed that granular nature of radiation from hot bodies was due to some property of the atoms producing it. Einstein extended his idea and postulated that packets or tiny bundles of energy are integral part of all electromagnetic radiation and that they could not be subdivided. These indivisible tiny bundles of energy he called photons. The beam of light with wavelength λ consists of stream of photons travelling at speed c and carries energy hf. From the theory of relativity momentum p of the photon is related to energy as

$$E = pc \quad \text{and} \quad E = hf \qquad \ldots\ldots \text{(i)}$$

$$\text{Thus} \quad pc = hf \quad \text{or} \quad p = \frac{hf}{c} = \frac{h}{\lambda} \qquad \ldots\ldots \text{(ii)}$$

The table relates the quanta emitted in different regions of the electromagnetic spectrum with energy. At the high end, γ-radiation with energy ~ 1 MeV is easily detected as quanta by a radiation detector and counter. At the other end, the energy of photon of radio waves is only about 10^{-10} eV. So millions of photons are needed to detect a signal and hence wave properties of radio waves predominate. The quanta are so close together in energy value that radio waves are detected as continuous radiation.



The emission or absorption of energy in steps may be extended to include any system such as a mass oscillating on a spring. However, the energy steps are far too small to be detected and so any granular nature is invisible. Quantum effects are only important when observing atomic sized objects, where h is a significant factor in any detectable energy change.

INTERACTION OF ELECTROMAGNETIC RADIATION WITH MATTER

Electromagnetic radiation or photon interact with matter in three different ways depending on their energy, the three processes are:

(i) Photoelectric effect

(ii) Compton effect

(iii) Pair production

Q.5 What is photoelectric effect? What are its experiment results? How Einstein explained it on the basis of quantum theory?

Ans. PHOTOELECTRIC EFFECT

The emission of electron from a metal surface when expose to light of suitable frequency is called **photoelectric effect**.

Demonstration of Photoelectric Effect

The photoelectric effect is demonstrated by the apparatus shown in the figure. An evacuated glass tube X contain two electrode. The electrode A is connected to positive terminal of the battery and is called anode. The electrode C is connected to –ve terminal of battery and is called cathode.

When monochromatic light is allowed to fall on cathode, it begins to emit electrons, these photoelectrons are attracted by the anode and the current is shown by ammeter. The current stops when light is cut off which shows emission of electrons with light.

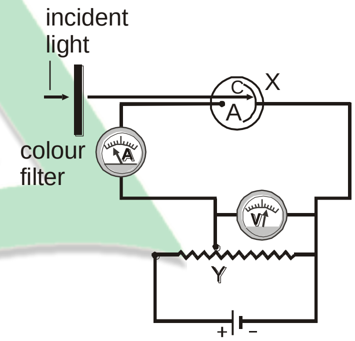


Fig. Experimental arrangement to observe photoelectric effect.

Calculation of Maximum Energy of Photoelectrons

The max energy of photoelectrons can be calculated by reversing the connection of battery in above circuit. The anode A is at –ve potential and cathode C is at +ve potential. In this condition, the photoelectrons emitted are repelled by anode A and photoelectric current decreases. If the potential is made more and more negative at a certain value of potential which is called **stopping potential**. The current just stops even the electron of max energy are not able to reach the collector plate C. The maximum energy of photoelectrons is given by

$$\frac{1}{2}mv_{max}^{2} = V_o e$$

where m is the mass and e is charge of electron. V_{max} is max velocity of electron and V_o is the stopping potential. The above experiment was repeated by changing frequency and intensity of incident light, and the following results were obtained

(1) There is minimum frequency (called threshold frequency) below which no electron are emitted whatever may be the intensity of light. The threshold frequency depends upon the nature of the metal.

(2) The electrons are emitted with different energies. The maximum energy of photoelectron depends upon particular metal surface and frequency of incident light. If the frequency is made different keeping the intensity as constant, the current is same but stopping potential is different as shown for different frequency which shows that maximum K.E of the photoelectrons is proportional to the frequency of incident light.

(3) The number of electron emitted is directly proportional to the intensity of light e.g., when light beam of higher intensity is made incident on metal surface the amount of current increase but current stops for same value of stopping potential V_o.

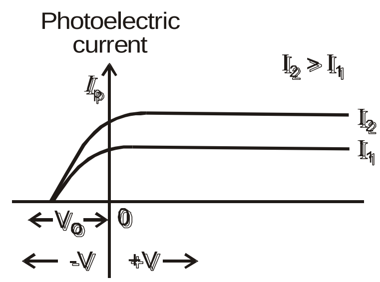


Fig. Characteristic curves of photo-current vs. applied voltage for two intensities of monochromatic light.

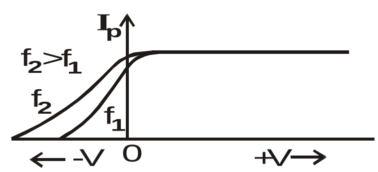


Fig. Characteristic curves of photo-current vs. applied voltage for light of different frequencies.

Quantum Theory of Light

Einstein using the idea of Maxwell Planck, proposed that light is emitted or absorbed in the form of quanta called **photons**. The energy of each photon is given by

$$E = hf$$

where h is the Planck's constant and f is the frequency of light.

Failure of Electromagnetic Theory to Explain Photoelectric Effect

According to electromagnetic theory of light, the increase of intensity of incident light should increase the K.E of emitted electron which experimentally is not correct. Also classical theory of light, fails to explain the threshold frequency.

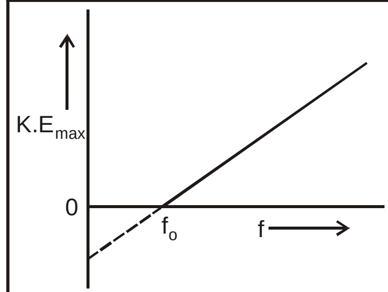
Explanation of Photoelectric Effect on the Basis of Quantum Theory

When photon falls on metal surface, it disappears after giving its energy to the electron of the metal. If this energy is more than the energy required to pull out an electron from the metal surface, the electron is instantaneously emitted. But if the energy is less than that value, no emission of electron takes place from the metal surface whatever may be the intensity of light. The minimum energy required to pull out an electron from metal surface is called **work function of metal surface** and is expressed as ϕ (work function) = hf.

Einstein Photoelectric Equation

Let a photon of energy "hf" falls on the metal surface. A part of photon energy is used to eject the electron from the metal surface and remaining energy is taken up by the electron in the form of K.E.

For Your Information



A graph of the maximum kinetic energy of photoelectrons vs. light frequency. Below a certain frequency, fo, no photoemission occurs.

Incident photon energy – Work function = K.E of electron

$$hf - hf_o = \text{K.E of electron}$$

$$hf - \phi = \frac{1}{2} mv^2_{max}$$

This is known as Einstein photoelectric equation.

If $(K.E)_{max} = 0$ then $f = f_o$ (threshold frequency).

Therefore $hf_o - \phi = 0$

$$hf_o = \phi$$

Hence Einstein equation can be written as

$$(K.E)_{max} = hf - hf_o$$

It is to be noted that all the emitted electrons do not possess the maximum kinetic energy, some electrons come straight out of the metal surface and some lose energy in atomic collisions before coming out. The above equation holds good only for those electrons which come out with full surplus energy.

Albert Einstein was awarded Nobel Prize in physics in 1921 for his explanation of photoelectric effect.

Note: The phenomenon of photoelectric effect cannot be explained if we assume that light consists of waves and energy is uniformly distributed over its wavefront. It can only be explained by assuming light consists of corpuscles of energy known as photon. Thus it shows the corpuscular nature of light.

Q.6 What is photocell?

Ans. **PHOTOCELL**

A photocell is based on photoelectric effect. A simple photocell is shown in figure. It consists of an evacuated glass bulb with a thin anode rod and a cathode of an appropriate metal surface. The material of the cathode is selected to suit to the frequency range of incident radiation over which the cell is operated. For example sodium or potassium cathode emits electrons for visible light, cesium coated oxidized silver emits electrons for infrared light and some other metals respond to ultraviolet radiation. When photo-emissive surface is exposed to appropriate light, electrons are emitted and a current flows in the external circuit which increases with the increase in light intensity. The current stops when the light beam is interrupted. The cell has wide range of applications. Some of these are to operate

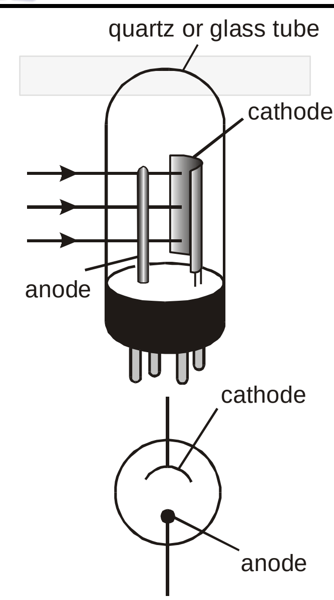


Fig. Simple photo-emissive cell

(1) Security systems

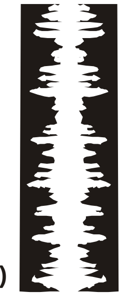
(2) Counting systems

(3) Automatic door systems

(4) Automatic street lighting

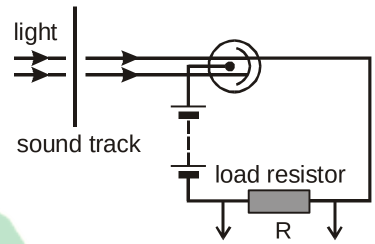
(5) Exposure meter for photography

(6) Sound track of movies



(a)

Fig. Sound track on a film which varies the intensity of light reaching the photo cell.



(b)

Fig. Photocell detection circuit for sound track of movies.

Q.7 Explain Compton effect.

Ans. **COMPTON EFFECT**

Arthur Holly Compton at Washington University in 1923 studied the scattering of X-rays by loosely bound electrons from a graphite target. He measured the wavelength of X-rays scattering at an angle θ with the original direction. He found that wavelength λ_s of the scattered X-rays is larger than the wavelength λ_i of the incident X-rays. This is known as Compton effect. The increase in wavelength of scattered X-rays could not be explained on the basis of classical wave theory. Compton suggested that X-rays consist of photons and in the process of scattering the photons suffer collision with electrons like billiard balls. In this collision, a part of incident photon energy and momentum is transferred to an electron. Applying energy and momentum conservation laws to the process, he derived an expression for the change in wavelength Δλ known as Compton shift for scattering angle θ as

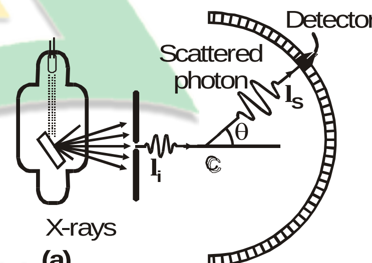


Fig. (a) Compton's scattering experiment.

$$\Delta\lambda = \frac{h}{m_oc}(1 - \cos\theta) \quad \dots \text{(i)}$$

where m_o is the rest mass of the electron. The factor $\frac{h}{m_oc}$ has dimensions of length and is called Compton wavelength and has the numerical value.

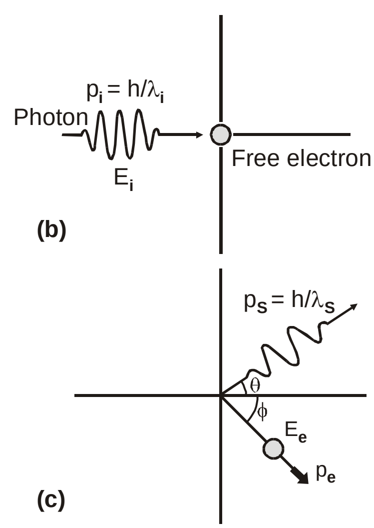


Fig. (b) A photon collides with an electron and (c) Both are scattered

$$\frac{h}{m_oc} = \frac{6.63 \times 10^{-34}\ \text{Js}}{9.1 \times 10^{-31}\ \text{kg} \times 3 \times 10^{8}\ \text{ms}^{-1}}$$

$$= 2.43 \times 10^{-12} \text{ m}$$

If the scattered X-ray photons are observed at $\theta = 90°$, the Compton shift $\Delta\lambda$ equals the Compton wavelength. The eq. (i) was found to be in complete agreement with Compton's experimental result, which again is a striking confirmation of particle like interaction of electromagnetic waves with matter.

Arthur Holly Compton was awarded Nobel Prize in physics in 1927 for his discovery of the effect named after him.

Q.8 Explain pair production and annihilation of matter.

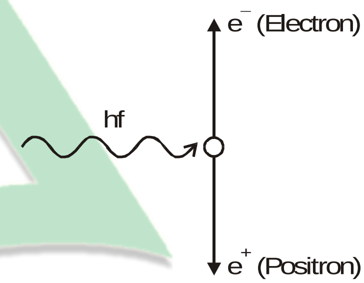
Ans.

PAIR PRODUCTION

If a very high energy photon such as γ-rays photon attempted to be slowed down by introducing heavy nucleus in its path, but without any physical interaction, the photon energy is changed into an electron position pair.

The phenomenon of creation of electron and positron in the process of slowing down of an energetic photon is called pair production or materialization of energy.

Positron is a positive electron. The creation of two particles with equal and opposite charge is essential for charge conservation. Positron is also called anti-electron. The process of pair production takes place in the electric field close to the nucleus. This process also takes place according to Einstein mass energy equation that is $E = mc^2$. For an e′ or position, the rest mass energy is equal to 0.511 MeV. Thus to create two positron, 2 × 0.511 MeV or 1.02 MeV energy is required. So, pair production takes place when photon energy $hf > 2m_oc^2$ or $hf > 1.02$ MeV is used pair and the remaining energy is taken up by the pair in the form of K.E. So,



$$\text{Energy of incident photon} = 2m_oc^2 + (K.E)_{e^-} + (K.E)_{e^+}$$

$$hf = 2m_oc^2 + (K.E)_{e^-} + (K.E)_{e^+}$$

ANNIHILATION OF MATTER

The phenomenon in which electron-positron pair combine together to form two γ-rays photon is called annihilation of matter.

(OR)

The reverse process of pair production is called annihilation of matter.

The two γ-ray photons produced move in the opposite direction, so, that momentum is conserved. Each photon has energy 0.511 MeV equivalent to rest mass energy of a particle.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

Each particle has antiparticle. For example, proton has antiparticle antiproton. Similarly, antineutron and so on, when particle and antiparticle combine together. They annihilate each other to form another form of energy. The existence of positron was predicted by Dirac in 1928 and it was discovered in the cosmic radiation in 1932 by Carl Anderson. It gradually became clear that every particle has a corresponding antiparticle with the same mass and charge (if it is a charged particle) but of opposite sign. Particles and antiparticles also differ in the signs of other quantum numbers that we have not yet discussed. A particle and its antiparticle cannot exist together at one place. Whenever they meet, they annihilate each other. That is, the particle disappear, their combined rest energies appear in other forms. Proton and antiproton annihilation has also been observed at Lawrence Berkeley Laboratory.

Q.9 *Describe the wave nature of particles. Also discuss Davisson and Germer experiment.*

Ans. WAVE NATURE OF PARTICLES

It has been observed that light shows a dual nature. It acts as waves in interference and diffraction and particle nature in photoelectric and Compton effect.

Loius de-Broglie proposed in 1924 that particles like electrons or protons should also possessed wave like particle. As momentum of photon is given by

$$P = \frac{h}{\lambda}$$

de-Broglie suggested that the momentum of particle of mass m moving with velocity v may also be given by similar expression

$$P = \frac{h}{\lambda} = mv$$

$$mv = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{mv}$$

Do You Know?

[illegible]

This is known as de-Broglie equation. It gives wavelength λ associated with a particle of mass m moving with velocity v.

Example

An object of large mass and ordinary speed has such a small wavelength that its wave effects such as interference and diffraction are negligible. When a rifle bullet of mass 20 g and flying with speed 330 ms^{-1} will have a wavelength λ given by

$$\lambda = \frac{\quad}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{2 \times 10^{-2} \times 330}$$

$$= 1 \times 10^{-34}\ m$$

This wavelength is so small that it is not measurable or detectable by any of its effects.

On the other hand for an electron moving with a speed of $1 \times 10^{6}\ ms^{-1}$.

$$\lambda = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1 \times 10^{6}}$$

$$= 7 \times 10^{-10}\ m$$

This wavelength is in the X-rays range. Thus, diffraction effects for electrons are measurable whereas diffraction or interference effects for bullets are not.

Davisson and Germer Experiment

The de-Broglie equation was confirmed by Davison and Germer. They showed that electrons are diffracted from metal crystals in the same way as x-rays or any other wave. The apparatus used is shown in figure. The electrons are obtained by heating a filament and are accelerated in electron gun by potential difference applied between filament and anode of electron-gun.

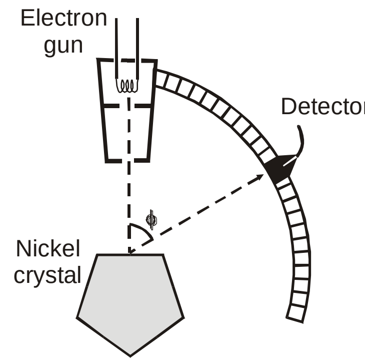


Fig. Experimental arrangement of Davisson and Germer for electron diffraction.

Thus,
$$\frac{1}{2}mv^2 = Ve$$
$$mv^2 = 2Ve$$
$$m^2v^2 = 2Vem$$
$$mv = \sqrt{2Vem}$$

Putting the value in de-Broglie equation

$$\lambda = \frac{h}{\sqrt{2Vem}}$$
$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 54 \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$
$$\lambda = 1.66 \times 10^{-10}\ \text{m}$$

The electron beam of energy Ve is made incident on a nickel crystal (Ni). The beam is diffracted and is recorded by moveable detector. The first order (n = 1) was obtained for an angle 65°. According to Bragg's equation

$$2d \sin\theta = n\lambda$$

where,
$$n = 1$$
$$\theta = 65°$$
$$d = 0.91 \times 10^{-10}\ \text{m}$$
$$2 \times 0.91 \times 10^{-10} \times \sin 65° = 1 \times \lambda$$
$$\lambda = 1.65 \times 10^{-10}\ \text{m}$$

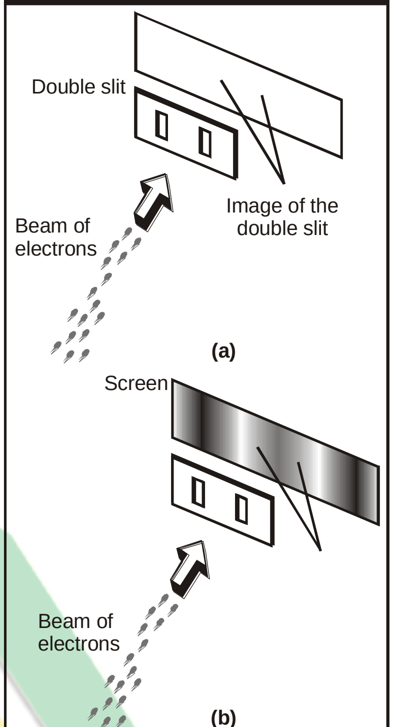
Thus experimental result of wavelength agrees with the theoretical result. The wave like property of diffraction pattern have also been observed for protons, neutrons, hydrogen atoms and helium atoms. For his work on the dual nature of particles, de-Broglie awarded a Nobel Prize in 1929 in physics Davison and Germer also shared the Nobel Prize in 1937 for this experimental conformation of the wave nature of particle.

Q.10 Describe the wave particle duality.

Ans. WAVE PARTICLE DUALITY

Interference and diffraction of light confirm its wave nature, while photoelectric effect proves the particle nature of light. Similarly, the experiments of Davisson and Germer and G.P. Thomson reveal wave like nature of electrons and in the experiment of J. J. Thomson to find e/m we had to assume particle like nature of the electron. In the same way we are forced to assume both wavelike and particle like properties for all matter electrons, neutrons, molecules etc. and also light, X-rays, γ-rays etc. have to be included in this. In other words, matter and radiation have a dual 'wave-particle' nature and this new concept is known as wave particle duality. Niels Bohr pointed out in stating his principle of complementarity that both wave and particle aspects are required for the complete description of both radiation and matter. Both aspects are always present and either may be revealed by an experiment. However, both aspects cannot be revealed simultaneously in a single experiment, which aspect is revealed simultaneously in a single experiment, which aspect is revealed is determined by the nature of the experiment being done. If you put a diffraction grating in the path of a light beam, you reveal it as a wave. If you allow the light beam to hit a metal surface, you need to regard the beam as a stream of particles to explain your observations. There is no simple experiment that you can carry out with the beam that will require you to interpret it has a wave and as a particle at the same time. Light behaves as a stream of photons when it interacts with matter and behaves as a wave in travelling from a source to the place where it is detected. In effect, all micro-particles (electrons, protons, photons, atoms etc.) propagate as if they were waves and exchange energies as if they were particles - that is the wave particle duality.

Do You Know?



(a) If electrons behaved as discrete particles with no wave properties, they would pass through one or the other of the two slits and strike the screen. Causing it to glow and produce exact images of the slits. (b) In reality the screen reveals a pattern of bright and dark fringes similar to light is used, and interference occurs between the light waves coming from each slit.

Use of Wave Nature of Particles

The fact that energetic particles have extremely short de-Broglie wavelengths has been put to practical use in many ultra-modern devices of immense importance such as electron microscope.

Q.11 Write a note on electron microscope.

Ans. ELECTRON MICROSCOPE

Electron microscope makes practical use of the wave nature of electrons which is thousands of time shorter than visible light which enables the electron microscope to distinguish details not visible with optical microscope. In an electron microscope, electric and magnetic fields rather than optical lenses are used to focus electrons by means of electromagnetic forces that are exerted on moving charges. The resulting deflections of the electrons beams are similar to the refraction effects produced by glass lenses used to focus light in optical microscope. The electrons are accelerated to high energies by applying voltage from 30 kV to several megavolts. Such high voltages give extremely short wavelength and also give the electron sufficient energy to penetrate specimen of reasonable thickness. A resolution of 0.5 to 1 nm is possible with a 50 kV microscope as compared to best optical resolution of 0.2 μm. A schematic diagram of the electron microscope is shown in the figure. The magnetic conducting lens concentrates the beam from an electron gun on to the specimen. Electrons are scattered out of the beam from the thicker parts of the specimen. The transmitted beam therefore has spatial differences in density that correspond to the features of the specimen. The objective and intermediate lenses produce a real intermediate image and projection lens forms the final image which can be viewed on a fluorescent screen or photographed on special film known as electron micrograph. A three dimensional image of remarkable quality can be achieved by modern versions called scanning electron microscopes.

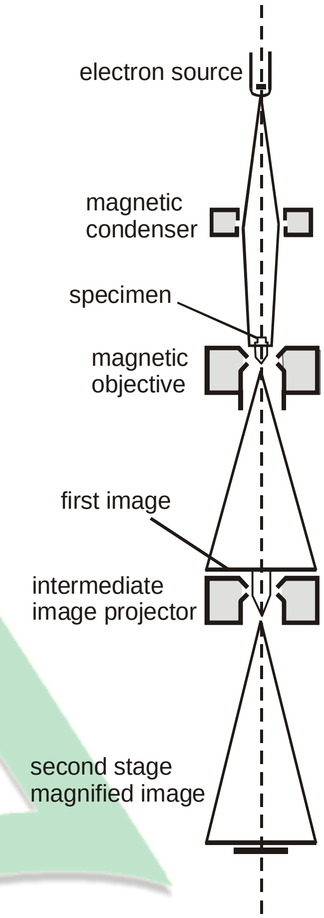


Fig. Electron microscope (Block diagram)

Do You Know

Q.12 State and prove the Heisenberg uncertainty principle.

Ans. UNCERTAINTY PRINCIPLE

This principle states that "it is impossible to measure the momentum and position of particle simultaneously with perfect accuracy". The uncertainty in momentum and position measurements is not due to measuring instrument but is related with wave particle duality of matter and radiation. This uncertainty is completely negligible for measurements of positions and momentum of macroscopic particles but is significant in the atomic particles. For examples a stream of light of photons scattering from a flying tennis ball effects its path. But one photon strikes an electron alters its position very much. Since, light has also wave properties, the accuracy of measurement of position is always of the order of one wavelength. Thus in order to measure position of an electron with less uncertainty and to avoid diffraction effect, light of very short wavelength must be used. Hence, for a microscopic particles moving along x-axis, the value of uncertainty in position Δx is of the order of wavelength λ used. So Δx can be expressed as

$$\Delta x \simeq \lambda$$

The light with short wavelength have large momentum. So, uncertainty in momentum of electron is increased. The uncertainty in the momentum of electron can be of the order of momentum of photon. So, the uncertainty in momentum of particle or electron along x-axis can be expressed as

$$\Delta P \simeq \frac{h}{\lambda}$$

For Your Information

[illegible]

Multiplying the above relations

$$\Delta P \,.\, \Delta x \simeq \frac{h}{\lambda} \,.\, \lambda$$

$$\Delta P \,.\, \Delta x \simeq h$$

This is the mathematical form of uncertainty principle. It states that the product of the uncertainty Δx in the position of a particle at some instant and the uncertainty ΔP in the x-component of its momentum at the same instant approximately equals Planck's constant h.

There is another form of uncertainty principle which relates the energy of a particle and the time at which it had the energy. If ΔE is the uncertainty in energy and if Δt is the time interval during which the particle had the energy.

then $$\Delta E \,.\, \Delta t \approx h$$

Thus more accurately we determined the energy of a particle, the more uncertain we will be of the time during which it has that energy.

According to Heisenberg's more careful calculations, he found that at the very best

$$\Delta x \,.\, \Delta p \geq \hbar$$

and $$\Delta E \,.\, \Delta t \geq \hbar$$

where $$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\ \text{Js}$$

Werner Karl Heisenberg received Nobel Prize for physics in 1932 for the development of quantum mechanics.

SOLVED EXAMPLES

EXAMPLE 19.1

The period of pendulum is measured to be 3.0 sec. in the inertial reference frame of the pendulum. What is its period measured by an observer moving at a speed of 0.95 c with respect to the pendulum?

Data

Proper time = t_o = 3.0 s

Velocity = v = 0.95 c

To Find

Time increased = t = ?

SOLUTION

Using

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{3.0}{\sqrt{1 - \frac{(0.95\ c)^2}{c^2}}}$$

$$= 9.6 \text{ sec.}$$

Result

Time increased = t = 9.6 sec.

EXAMPLE 19.2

A bar 1.0 m in length and located along x-axis moves with a speed of 0.75 c with respect to a stationary observer. What is the length of the bar as measured by the stationary observer?

Data

Proper length = l_o = 1.0 m

Velocity = v = 0.75 c

To Find

Length contraction = l = ?

SOLUTION

Using

$$l = l_o\sqrt{1-\frac{v^2}{c^2}}$$

$$= 1.0\sqrt{1-\frac{(0.75\ c)^2}{c^2}}$$

$$= 0.66\ \text{m}$$

Result

Length contraction = l = 0.66 m

EXAMPLE 19.3

Find the mass m of a moving object with speed 0.8 c.

Data

Speed of object = v = 0.8 c

To Find

Mass of object = m = ?

SOLUTION

Using

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{m_o}{\sqrt{1-\frac{(0.8\ c)^2}{c^2}}}$$

$$= 1.67\ m_o$$

Result

Mass of object = m = 1.67 m_o

EXAMPLE 19.4

Assuming you radiate as does a black body at your temperature about 37°C, at what wavelength do you emit the most energy?

Data

Temperature = T = 37°C + 273

= 310 K

Wien's constant = 2.9×10^{-3} mK

To Find

Maximum wavelength = λ_{max} = ?

SOLUTION

Using

$$\lambda_{max} \times T = \text{Wien's constant}$$

$$\lambda_{max} = \frac{\text{Wien's constant}}{T}$$

$$= \frac{2.9 \times 10^{-3}}{310}$$

$$= 9.35 \times 10^{-6} \text{ m}$$

$$\lambda_{max} = 9.35 \ \mu\text{m}$$

Result

Maximum wavelength $= \lambda_{max} = 9.35 \ \mu\text{m}$

EXAMPLE 19.5

What is the energy of a photon in a beam of infrared radiation of wavelength 1240 nm?

Data

Wavelength $= \lambda = 1240$ nm

$= 1240 \times 10^{-9}$ m

To Find

Energy $= E = ?$

SOLUTION

Using

$$E = hf$$

But $$f = \frac{c}{\lambda}$$

$$\boxed{E = \frac{hc}{\lambda}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1240 \times 10^{-9}}$$

$$= 1.6 \times 10^{-19} \text{ J}$$

$$E = 1.0 \text{ eV}$$

Result

Energy $= 1.0$ eV

EXAMPLE 19.6

A sodium surface is illuminated with light of wavelength 300 nm. The work function of sodium metal is 2.46 eV. Find

(a) Maximum energy of the ejected electron.

(b) Determine the cutoff wavelength for sodium.

Data

Wavelength $= \lambda = 300\ \text{nm}$

$= 300 \times 10^{-9}\ \text{m}$

Work function $= \phi = 2.46\ \text{eV}$

$= 2.46 \times 1.6 \times 10^{-19}\ \text{J}$

To Find

(a) Maximum K.E. = ?

(b) Cutoff wavelength λ_o = ?

SOLUTION

(a) For maximum K.E of ejected electron

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9}}$$

$$= 6.63 \times 10^{-19}\ \text{J}$$

$$E = 4.14\ \text{eV}$$

$$K.E_{max} = hf - \phi$$

$$= 4.14 - 2.46$$

$$= 1.68\ \text{eV}$$

(b) Now

$$\phi = \frac{\ }{\lambda_o}$$

$$\lambda_o = \frac{hc}{\phi}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{3.94 \times 10^{-19}}$$

$$= 5.05 \times 10^{-7}\ \text{m}$$

$$= 505 \times 10^{-9}$$

$$= 505\ \text{nm}$$

Result

(a) $(K.E)_{max} = 1.68\ \text{eV}$

(b) Cut off wavelength = 505 nm

EXAMPLE 19.7

A 50 KeV photon is Compton scattered by a quasi-free electron. If the scattered photon comes off at 45° what is its wavelength?

Data

Energy $= E = 50$ KeV

$= 50 \times 10^{3}$ eV

$= 5 \times 10^{4} \times 1.6 \times 10^{-19}$ J

Scattering angle $= \underline{\theta} = 45°$

To Find

Scattering wavelength $= \lambda_s = ?$

SOLUTION

Using

$$E = \frac{hc}{\lambda}$$

$$\lambda_i = \frac{hc}{E}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{5 \times 10^{4} \times 1.6 \times 10^{-19}}$$

$$= \frac{19.89 \times 10^{-26}}{8 \times 10^{-15}}$$

$$= 2.486 \times 10^{-26 + 15}$$

$$= 2.486 \times 10^{-11} \text{ m}$$

= 0.0248

$$\lambda_i = 0.0248 \text{ nm}$$

Now using

$$\Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta)$$

$$\lambda_s - \lambda_i = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 45°)$$

$$= \frac{6.63 \times 10^{-34 - 8 + 31}}{27.3}(1 - 0.707)$$

$$= 0.2429 \times 10^{-11}(0.293)$$

$$= 0.07 \times 10^{-11}$$

$$= 0.0007 \times 10^{-9} \text{ m}$$

$$= 0.0007 \text{ nm}$$

$$\therefore \quad \lambda_s = \lambda_i + 0.0007$$

$$= 0.0248 + 0.0007$$

$$= 0.0255 \text{ nm}$$

Result

$$\lambda_s = 0.0255 \text{ nm}$$

EXAMPLE 19.8

A particle of mass 5 mg moves with speed 8.0 m/s. Calculate its de-Broglie wavelength.

Data

Mass of particle = m = 5 mg

= 5×10^{-6} kg

Speed = v = 8 m/s

To Find

Wavelength = λ = ?

SOLUTION

Using

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{5 \times 10^{-6} \times 8.0}$$

$$= \frac{6.63}{40} \times 10^{-28}$$

$$\lambda = 1.66 \times 10^{-29} \text{ m}$$

Result

Wavelength = λ = 1.66×10^{-29} m

EXAMPLE 19.9

An electron is accelerated through a potential difference of 50 V. Calculate its de-Broglie wavelength.

Data

Potential difference = V = 50 V

To Find

$$\lambda = ?$$

SOLUTION

Using

$$\lambda = \frac{h}{\sqrt{2mVe}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 50 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.74 \times 10^{-10}\ \text{m}$$

Result

$$\lambda = 1.74 \times 10^{-10}\ \text{m}$$

EXAMPLE 19.10

The life time of an electron in an excited state is about 10^{-6} s. What is its uncertainty in energy during this time?

Data

$$\Delta t = 10^{-8}\ \text{s}$$

To Find

$$\Delta E = ?$$

SOLUTION

Using

$$\Delta E\, \Delta t \simeq \hbar$$

$$\therefore \quad \Delta E = \frac{\hbar}{\Delta t}$$

$$= \frac{1.05 \times 10^{-34}}{10^{-8}}$$

$$\Delta E = 1.05 \times 10^{-26}\ \text{J}$$

Result

$$\Delta E = 1.05 \times 10^{-26}\ \text{J}$$

EXAMPLE 19.11

An electron is to be confined to a box of size of the nucleus (1×10^{-14} m). What would the speed of electron be if it were so confined?

Data

$$\Delta x = 1 \times 10^{-14}\ \text{m}$$

To Find

$$\Delta V = ?$$

SOLUTION

Using

$$\Delta P \approx \frac{\hbar}{\Delta x}$$

$$m\,\Delta V = \frac{\hbar}{\Delta x}$$

$$\Delta V = \frac{\hbar}{m\Delta x}$$

$$= \frac{1.05 \times 10^{-34}}{9.1 \times 10^{-31} \times 1 \times 10^{-14}}$$

$$= \frac{1.05}{9.1} \times 10^{-34+31+14}$$

$$= 0.115 \times 10^{11}$$

$$\Delta V = 1.15 \times 10^{10}\ \text{m/s}$$

Result

$$\Delta V = 1.15 \times 10^{10}\ \text{m/s}$$