Chapter 12

ELECTROSTATICS

LEARNING OBJECTIVES

At the end of this chapter the students will be able to:

- Understand and describe Coulomb's law.
- Describe that a charge has a field of force around it.
- Understand fields of like and unlike charges.
- Explain the electric intensity in a free space and in other media.
- State and prove Gauss's law.
- Appreciate the applications of Gauss's law.
- Relate electric field strength and potential gradient.
- Find expression for potential at a point due to a point charge.
- Describe and derive the value of electric charge by Milikan's method.
- Calculate the capacitance of parallel plate capacitor.
- Understand and describe electric polarization of dielectric.
- Find energy expression of a charged capacitor.

Q.1 Define electrostatics.

Ans. **ELECTROSTATICS**

It is the branch of physics which deals with charges at rest, under the action of electric forces.

Q.2 Describe electric charges.

Ans. **ELECTRIC CHARGE**

It is the property of a material due to which it attracts or repels other bodies. In 1600 A.D. Gilbert showed that some of the substance when rubbed with other acquired the property of attraction.

A similar property is observed when a glass rod is rubbed with silk or ebonite rod is rubbed with animal fur, they acquired the property of attracting the small bodies Franklin gave the name of positive charge on a glass rod rubbed with silk and negative charge on the ebonite rod rubbed with animal fur.

According to modern concept, each substance consists of atoms, the atom has a central part called the nucleus a round which the electrons revolve. The nucleus is made up of protons and neutrons. The proton has a positive charge and the electron has a negative charge. So the atom on the whole is neutral. The total positive charge on the nucleus is equal to the total negative charge on the electrons. Thus when one metal rubbed with another, sharing of electrons take place. That is one loses electrons and other gains electrons. When a glass rod rubbed with silk, the rod loses electrons and silk gains electron. As both were initially neutral. So the glass rod loses electrons gets positive charge and the silk gains electron, gets negative charge.

Unit of Electric Charge

SI unit of charge is coulomb.

Q.3 State and explain coulomb's law.

Ans. COULOMB'S LAW

Introduction

There are two kinds of charges, positive and negative charges. The qualitative investigations made in previous classes revealed that similar charges repel and opposite charges attract each other, with a force known as force of interaction. To find magnitude and direction of such electrostatic interaction between charges coulomb carried out series of experiments using an apparatus known as **Torsion Balance** and proposed a law as follows.

Statement

This law states that "the force of attraction or repulsion between two point charges is directly proportional to the product of magnitude of charges and inversely proportional to the square of the distance between their centers".

Explanation

Consider two point charges q_1 and q_2 having a distance r between them. If F is the force, then according to Coulomb's law

$$F \propto q_1q_2 \qquad \text{...... (i)}$$

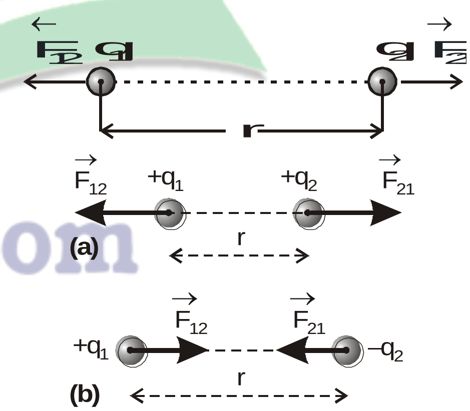
$$F \propto \frac{1}{r^2} \qquad \text{...... (ii)}$$

Combining (i) and (ii)

$$F \propto \frac{q_1q_2}{r^2}$$

$$F = K\frac{q_1q_2}{r^2}$$

Which is magnitude of coulomb force between charges. Where K is the constant of proportionality and its value is 9×10^9 Nm^2/C^2. Its value depends upon the nature of the medium between the two charges and system of units in which F, q and r are measured. If the medium between the two point charges is free space then



like charges and (b) attractive forces between unlike charges.

$$K = \frac{1}{4\pi\epsilon_o}$$

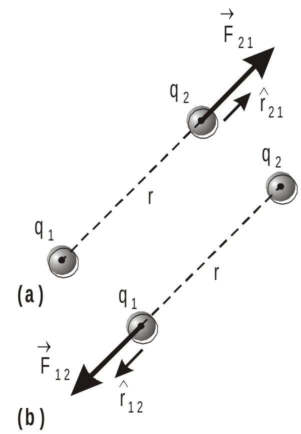
where ϵ_o is an electrical constant, known as permitivity of free space. "The permitivity of free space or air is the permission given by air for the transmission of force from one charge to other charge". In SI units, its value is 8.85×10^{-12} $C^2N^{-1}m^{-2}$. Coulomb's force always acts along the line joining two charges. Therefore Coulomb's force in free space is

$$F = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}$$

Q.4 What is the vector form of Coulomb's law?

Ans. **VECTOR FORM OF COULOMB'S FORCE**

Consider two like charges q_1 and q_2 having a distance r between them. If we denote the force exerted on q_2 by q_1 as $\vec{F}_{21}$ and that on charge q_1 due to q_2 as $\vec{F}_{12}$.



If $\hat{r}_{21}$ is a unit vector directed from q_1 to q_2 and $\hat{r}_{12}$ is the unit vector directed from q_2 to q_1 then

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{21} \quad \ldots\ldots \text{(i)}$$

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

From figure

$$\hat{r}_{21} = -\hat{r}_{12}$$

Putting this in equation (i)

$$\therefore \quad \vec{F}_{21} = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r_2}(-\hat{r}_{12})$$

$$\vec{F}_{21} = -\left(\frac{1}{4\pi\epsilon_o}\cdot\frac{q_1q_2}{r_2}\hat{r}_{12}\right)$$

$$\therefore \quad \vec{F}_{21} = -\vec{F}_{12}$$

Hence Coulombs force is mutual force, it means that if q_1 exerts a force on q_2 then q_2 also exerts an equal and opposite force on q_1. Coulombs force is also known as electrostatic force and force of interaction.

Q.5 What is the effect of medium on the Coulomb's force?

Ans. **EFFECT OF MEDIUM BETWEEN THE TWO CHARGES UPON THE COULOMB'S FORCE**

If the medium is an insulator, it is usually referred as dielectric. It has been found that the presence of a dielectric always reduces the electrostatic force as compared with that in free space by a certain factor which is a constant for the given dielectric. This constant is known as relative permitivity i.e., ϵ_r. Thus the Coulomb's force in a medium of relative permitivity ϵ_r is given by

$$F_{med} = \frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q_1q_2}{r^2} \quad \ldots\ldots \text{(i)}$$

when air is placed between the same two charges then,

$$F_{air} = \frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r^2} \quad \ldots\ldots \text{(ii)}$$

Dividing (ii) by (i)

$$\frac{F_{air}}{F_{med}} = \frac{\frac{1}{4\pi\epsilon_o}\frac{q_1q_2}{r_2}}{\frac{1}{4\pi\epsilon_o\epsilon_r}\frac{q_1q_2}{r_2}}$$

TABLE

Material	ε_r
Vacuum	1
Air (1 atm)	1.0006
Ammonia (liquid)	22–25
Bakelite	5–18
Benzene	2.284
Germanium	16
Glass	4.8–10
Mica	3–7.5

$$= \frac{1}{\frac{1}{\epsilon_r}}$$

$$\frac{F_{air}}{F_{med}} = \epsilon_r$$

$$F_{med} = \frac{F_{air}}{\epsilon_r}$$

Paraffined paper	2
Plexiglas	3.40
Rubber	2.94
Teflon	2.1
Transformer oil	2.1
Water (distilled)	78.5

As for all dielectrics $\epsilon_r > 1$ except for air which is 1. i.e., $\epsilon_r = 1.0.006$ this value is close to one that with negligible error.

$$\therefore \quad F_{med} < F_{air}$$

Relative Permitivity

It is the ratio of force between the two charges placed in air to the force between the same two charges when a dielectric is placed between them. It has no unit.

Q.6 Describe the fields of force.

Ans. **FIELDS OF FORCE**

Newton's universal gravitational law and Coulomb's law enable us to calculate the magnitude as well as the directions of the gravitational and electric forces, respectively. However one may question:

(a) What are the origins of these forces?

(b) How are these force transmitted from one mass to another or from one charge to another?

The answer to (a) is still unknown; the existence of these forces is accepted as it is. That is why they are called **basic forces of nature**.

Faraday Concept:

To describe the mechanism by which electric force is transmitted, Michael Faraday (1791-1867) introduced the concept of an electric field. According to his theory, it is the intrinsic property of nature that an electric field exists in the space around an electric charge. This electric field is considered to be a force field that exerts a force on other charges placed in that field. **For example**, a charge q produces an electric field in the space surrounding it. This field exists whether the other charges are present in space or not. However, the presence of field cannot be tested until another charge q_o is brought into the field. Thus the field of charge q interacts with q_o to produce an electrical force. The interaction between q and q_o is accomplished in two steps: (a) the charge q produces a field and (b) the field interacts with charge q_o to produce a force **F** on q_o. These two steps are illustrated in figure as shown.

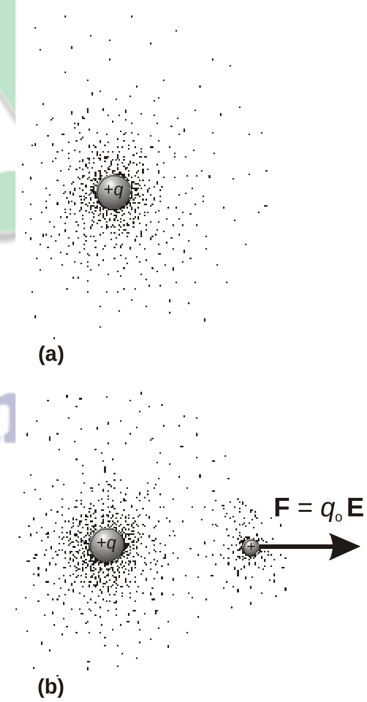


Fig. (a) Dots surrounding the positive charge indicate the presence of the electric field. The density of the dots is proportional to the strength of the electric field at different points. (b) Interaction of the field with the charge q_o.

In this figure the density of dots is proportional to the strength of the field at the various points.

Q.7 Define electric field and electric field intensity. Also calculate the electric field intensity due to a point charge.

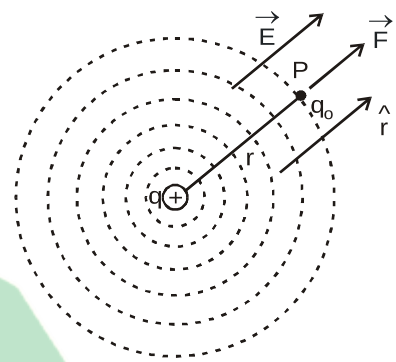
Ans. ELECTRIC FIELD

The region or space around a charge within which other charges are influenced is called electric field.

Electric Field Intensity

Electric field intensity $\vec{E}$ at a point is force per unit charge acting on a positive test charge placed at that point.

It is a vector quantity and its direction is same as that of force $\vec{F}$. Let $\vec{F}$ is the force experienced by test charge q_o placed in the field of a charge q then,



$$\vec{E} = \frac{\vec{F}}{q_o}$$

Unit of Electric Intensity

SI unit of electric intensity is NC^{-1} or $\frac{N}{C}$.

Electric Field Intensity Due to a Point Charge

Consider a point charge q placed at O. Now when a positive test charge q_o is placed inside the field of charge q at point P which is at a distance r from O, then according to Coulomb's law, the force experienced by q_o is

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{r^2}\hat{r} \qquad \text{...... (i)}$$

where $\hat{r}$ is a unit vector directed from point charge q to the test charge q_o which is placed at P where $\vec{E}$ is to be evaluated.

As $$\vec{E} = \frac{F}{q_o} \qquad \text{...... (ii)}$$

Putting the value of $\vec{F}$ in eq. (ii)

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{qq_o}{q_o r^2}\hat{r}$$

$$= \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$

where $\hat{r}$ is a unit vector directed from the point charge q to the test charge q_o.

The magnitude of electric intensity is

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}$$

$$E = \text{Constant } \frac{q}{r^2}$$

$$E \propto \frac{1}{r^2}$$

Thus electric intensity is inversely proportional to the square of distance between the charges.

Q.8 Describe the electric field lines. Write some properties of electric lines of force.

Ans. ## ELECTRIC FIELD LINES

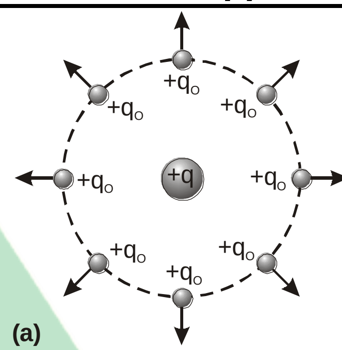
Definition

"The number of lines per unit area passing perpendicularly through an area is proportional to the magnitude of the electric field".

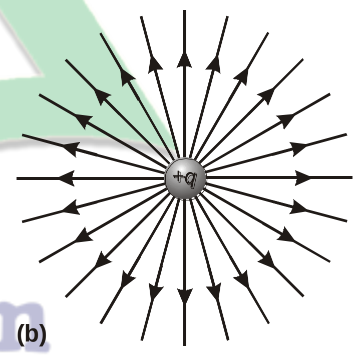
Explanation

A visual representation of the electric field can be obtained in terms of electric field lines; an ideal proposed by Michael Faraday. Electric field lines can be thought of a "map" that provides information about the direction and strength of the electric field at various places. As electric field lines provide information about the electric force exerted on a charge, the lines are commonly called "lines of force".

To introduce electric field lines, we place positive test charges each of magnitude q_o at different places but at equal distances from a positive charge +q as shown in the figure. Each test charge will experience a repulsive force, as indicated by arrows in figure (a). Therefore, the electric field created by the charge +q is directed radially outward. Figure (b) shows corresponding field lines which show the field direction. Figure shows the electric field lines in the vicinity of a negative charge –q. In this case the lines are directed radially "inward", because the force on a positive test charge is now of attraction, indicating the electric field points inward.

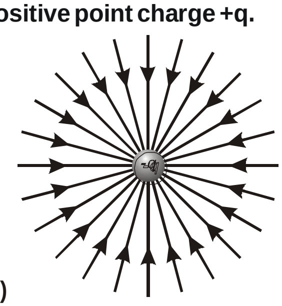


(a)



(b)

Fig. (a) A positive test charge $+q_o$ placed anywhere in the vicinity of a positive point charge +q, experiences a repulsive force directed radially outward from the positive point charge +q.



(c)

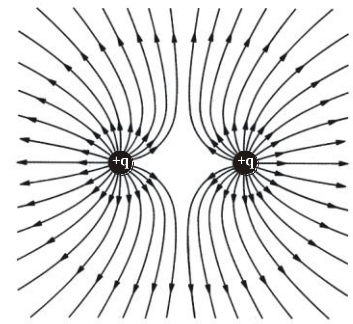
Fig. The electric field lines are directed radially inward towards a negative point charge –q.

Figures represent two dimensional pictures of the field lines. However, electric field lines emerge from the charges in three dimensions, and an infinite number of lines could be drawn.

The electric field lines "map" also provides information about the strength of the electric field. As we notice in figures that field lines are closer to each other near the charges where the field is strong while they continuously spread out indicating a continuous decrease in the field strength.

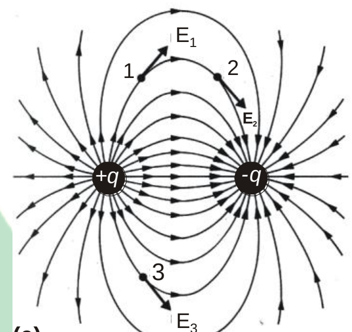
The electric field lines are curved in case of two identical separated charges. Figure (d) shows the pattern of lines associated with two identical positive point charges of equal magnitude. It reveals that the lines in the region between two like charges seem to repel each other. The behaviour of two identical negatively charges will be exactly the same. The middle region shows the presence of a zero field spot of neutral zone.

The figure (c) shows the electric field pattern of two opposite charges of same magnitudes. The filed lines start from positive charge and end on a negative charge. The electric field at points such as 1, 2, 3 is the resultant of fields created by the two charges at these points. The directions of the resultant intensities is given by the tangents drawn to the field lines at these points.



(d)

Fig. The electric field lines for two identical positive point charges.



(e)

Fig. Attractive forces between unlike charges.

In the regions where the field lines are parallel and equally spaced, the same number of lines pass per unit area and therefore, field is uniform on all points. Figure (f) shows the field lines between the plates of a parallel plate capacitor. The field is uniform in the middle region where field lines are equally spaced.

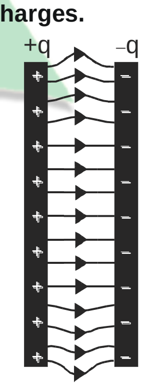
We are now in a position to summarize the properties of electric field lines.

(1) Electric field lines originate from positive charges and end on negative charges.

(2) The tangent to a field line at any point gives the direction of the electric field at that point.

(3) The lines are closer where the field is strong and the lines are farther apart where the field is weak.

(4) No two lines cross each other. This is because $\vec{E}$ has only one direction at any given point. If the lines cross, $\vec{E}$ could have more than one direction.



(f)

Fig. In the central region of a parallel plate capacitor the electric field lines are parallel that the electric field there has the same magnitude and direction at all points.

APPLICATIONS OF ELECTROSTATICS

There are two applications of electrostatics

(1) Xerography (Photocopier)

(2) Inkjet Printers

Q.9 Describe xerography in detail.

Ans.

XEROGRAPHY (PHOTOCOPIER)

Figure describes a photocopy machine.

Definition

The copying process is called xerography, from the Greek word “xeros” and “graphos”, meaning “dry writing”.

Construction

The heart of machine is a drum which is an aluminium cylinder coated with a layer of selenium. Aluminium is an excellent conductor. On the other hand, selenium is an insulator in the dark and becomes a conductor when exposed to light; it is a photoconductor. As a result, if a positive charge is sprinkled over the selenium it will remain there as long as it remains in dark. If the drum is exposed to light, the electrons from aluminium pass through the conducting selenium and neutralize the positive charge.

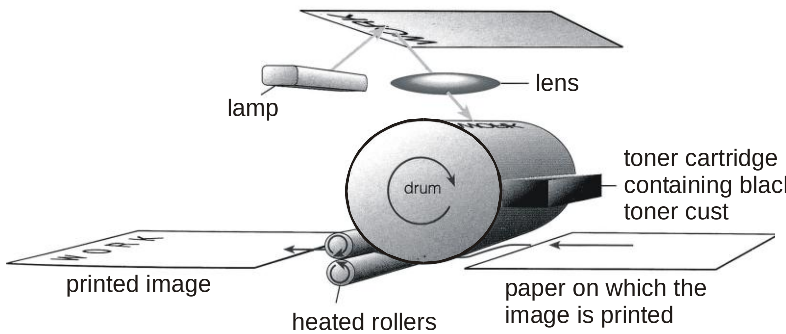
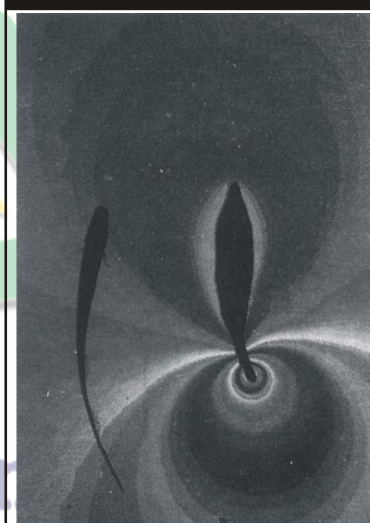


Fig. The basics of photocopying. The lamp transfers an image of the page to the drum, which leaves a static charge. The drum collects toner dust and transfers it to the paper. The toner is melted onto the page.

Working

If the drum is exposed to an image of the document to be copied, the dark and light areas of the document produce corresponding areas on the drum. The dark areas retain their positive charge, but light areas become conducting, lose their positive charge and become neutral. In this way, a positive charge image of the document remains on the selenium surface. Then a special dry, black powder called "toner" is given a negative charge and spread over the drum, where it sticks to the positive charged areas. The toner from the drum is transferred on to a sheet of paper on which the document is to be copied. Heated pressure rollers then melt the toner into the paper which is also given an excess positive charge to produce the permanent impression of the document.

For Your Information


This computer image shows the electric field lines generated by the fish at the top of the picture. Through the electric field, the presence of other fish can be silhouetted at the bottom.

Q.10 What is the inkjet printers?

Ans. **INKJET PRINTERS**

An inkjet printer figure (a) is a type of printer which uses electric charge in its operation. While shuttling back and forth across the paper, the inkjet printer "ejects" a thin stream of ink. The ink is forced out of a small nozzle and breaks up into extremely small droplets. During their flight, the droplets pass through two electrical components, a "charging electrode" and the "deflection plates" (a parallel plate capacitor). When the printhead moves over regions of the paper which are not to be inked,

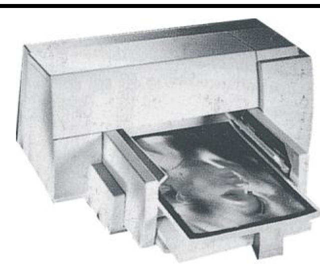


Fig. (a) An inkjet printer

the charging electrode is left on and gives the ink droplets a net charge. The deflection plates divert such charged drops into a gutter and in this way such drops are not able to reach the paper. Whenever ink is to be placed on the paper, the charging control, responding to computer, turns off the charging electrode. The uncharged droplets fly straight through the deflection plates and strike the paper. Schematic diagram of such a printer is shown by figure (b). Inkjet printers can also produce coloured copies.

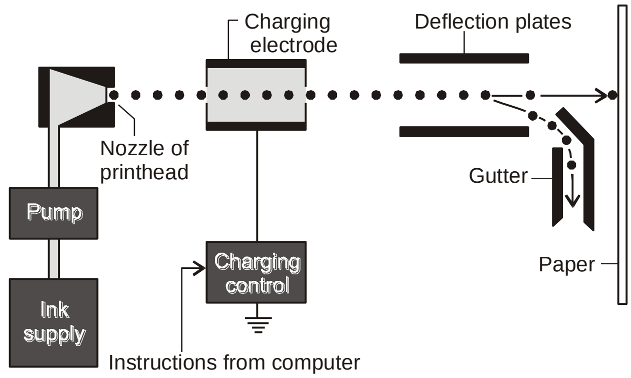


Fig. (a) An inkjet printhead ejects a steady flow of ink droplets. The charging electrodes are used to charge the droplets that are not needed on the paper. Charged droplets are deflected into a gutter by the deflection plates, while uncharged droplets fly straight onto the paper.

Vector Area

Usually the element of area is represented by a vector area $\vec{A}$ whose magnitude is equal to the surface area A of the element and whose direction is normal to the surface area.

Q.11 Define electric flux. Also discuss its particular cases.

Ans. **ELECTRIC FLUX**

The number of field lines passing through a certain element of area is known as electric flux through that area. It is denoted by Greek letter ϕ.

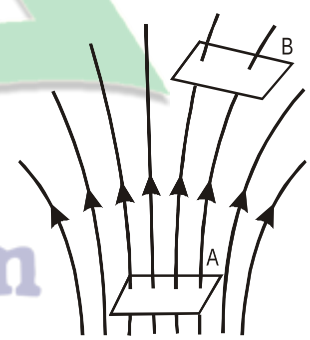


Fig. Electric flux through a surface normal to E.

Mathematical Definition

The electric flux ϕ_e through a patch of flat surface is defined as the scalar product of electric intensity $\vec{E}$ and vector area $\vec{A}$.

$$\therefore \qquad \phi_e = \vec{E} \cdot \vec{A}$$

$$\phi_e = EA\cos\theta$$

where θ is the angle between E and A. Electric flux, being a scalar product is a scalar quantity.

Unit of Electric Flux

SI unit of electric flux is Nm^2C^{-1}.

Special Cases

Case-I **When area element (plane of the surface) is held perpendicular to electric field $\vec{E}$. (OR vector area $\vec{A}$ is parallel to $\vec{E}$)**

As $\phi_e = \vec{E} \cdot \vec{A}$

$\phi_e = EA \cos\theta$

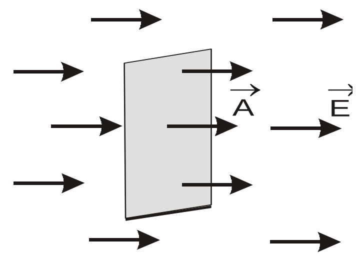
In this case $\theta = 0°$

$\phi_e = EA \cos 0°$

$\phi_e = EA\,(1)$

$\phi_e = EA$

In this case the flux is maximum.



Case-II **When area element (plane of surface) held parallel to $\vec{E}$. (Vector area $\vec{A}$ is perpendicular to $\vec{E}$)**

As $\phi_e = \vec{E} \cdot \vec{A}$

$\phi_e = EA \cos\theta$

In this case

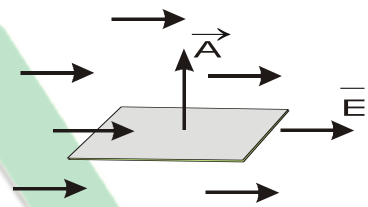
As $\theta = 90°$

$\phi_e = EA \cos 90°$

$\phi_e = EA\,(0)$

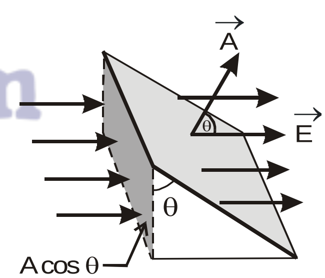
$\phi_e = 0$

In this case the flux is minimum.



Case-III In the case when A is neither perpendicular nor parallel to field lines but makes an angle θ with the lines. In this case we have to find the projection of the area which is perpendicular to the field lines. The projection of area is $A\cos\theta$. The flux ϕ_e in this case is

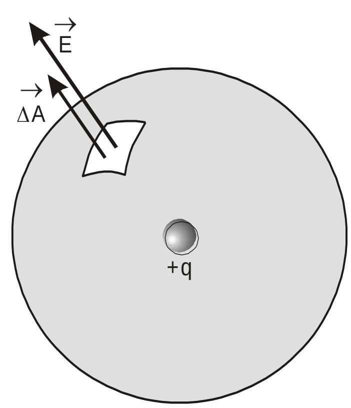
$\phi_e = EA \cos\theta$



Q.12 Calculate the electric flux through a surface enclosing a charge.

Ans. ELECTRIC FLUX THROUGH A SURFACE ENCLOSING A CHARGE

Consider a closed surface, in shape of a sphere of radius 'r' due to a point charge q placed at the center of sphere as shown in figure. To apply $\phi_e = \vec{E} \cdot \vec{A}$, the surface area should be flat. For this reason, the total surface area of the sphere is divided into 'n', small patches with areas of magnitudes $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, \ldots\ldots, \Delta\vec{A}_n$. If n is very large, each patch would be a flat element of area. The corresponding vector areas are $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, \ldots, \Delta\vec{A}_n$ respectively. The direction of each vector area is along perpendicular drawn outward to the corresponding



patch. The electric intensities at the centers of vector areas $\Delta\vec{A}_1, \Delta\vec{A}_2, \Delta\vec{A}_3, \ldots, \Delta\vec{A}_n$ are $\vec{E}_1, \vec{E}_2, \ldots, \vec{E}_n$ respectively.

The total flux passing through the closed surface is

$$\phi_e = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \ldots\ldots + \phi_n$$

$$\phi_e = \vec{E}_1 \cdot \Delta\vec{A}_1 + \vec{E}_2 \cdot \Delta\vec{A}_2 + \vec{E}_3 \cdot \Delta\vec{A}_3 \ldots\ldots \vec{E}_n \cdot \Delta\vec{A}_n$$

The direction of electric intensity and vector area is same at each patch.

i.e., $\theta = 0°$

$$\therefore \quad \phi_e = E_1\Delta A_1 \cos 0° + E_2\Delta A_2 \cos 0° + E_3\Delta A_3 \cos 0°$$

$$= E_1\Delta A_1 + E_2\Delta A_2 + E_3\Delta A_3 + \ldots\ldots + En\Delta A_n \quad (\because \cos 0° = 1)$$

The magnitude of electric intensity is same at the surface of each patch.

i.e., $|\vec{E}_1| = |\vec{E}_2| = |\vec{E}_3| \ldots\ldots |\vec{E}_n| = E$ because they are all equidistance from the charge.

$$\therefore \quad \phi_e = E\Delta A_1 + E\Delta A_2 + E\Delta A_3 + \ldots\ldots + E\Delta A_n$$

$$\phi_e = E(\Delta A_1 + \Delta A_2 + \Delta A_3 + \ldots\ldots + \Delta A_n)$$

As $\Delta A_1 + \Delta A_2 + \ldots\ldots + \Delta A_n$ = Total spherical surface area

$$= 4\pi r^2$$

and

$$E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}$$

Therefore,

$$\phi_e = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\text{(total spherical surface area)}$$

$$\phi_e = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2} \times (4\pi r^2)$$

$$\phi_e = \frac{q}{\epsilon}$$

Conclusion

So we conclude that total flux through a closed surface does not depend upon the shape or geometry of the closed surface but it depends upon the medium and the charge enclosed.

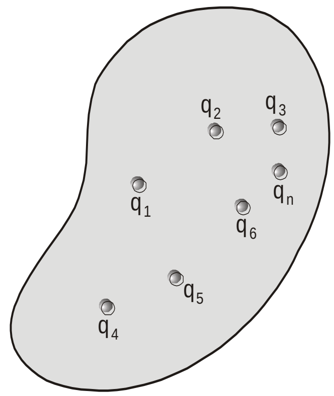
Q.13 State and explain Gauss's law.

Ans. GAUSS'S LAW

Statement

This law states that "the flux through any closed surface is $\frac{1}{\epsilon_o}$ time the total charge enclosed in it".

Proof



Suppose point charges $q_1, q_2, q_3, \ldots, q_n$ are arbitrarily distributed in a closed surface as shown in figure. As electric flux through a surface enclosing a charge is $\phi_e = \frac{q}{\epsilon_o}$. Hence the electric flux passing through a closed surface is

$$\phi_e = \phi_1 + \phi_2 + \phi_3 + \ldots\ldots + \phi_n$$

$$\phi_e = \frac{q_1}{\epsilon_o} + \frac{q_2}{\epsilon_o} + \frac{q_3}{\epsilon_o} + \ldots\ldots + \frac{q_n}{\epsilon_o}$$

$$\phi_e = \frac{1}{\epsilon_o}(q_1 + q_2 + q_3 + \ldots\ldots + q_n)$$

$$\phi_e = \frac{1}{\epsilon_o}(\text{total charge enclosed by the closed surface})$$

$$\phi_e = \frac{1}{\epsilon_o} \times Q$$

which is the mathematical form of Gauss's law.

APPLICATIONS OF GAUSS'S LAW

Gauss's law is applied to calculate the electric intensity due to different charge configurations. Steps to find electric intensity are:

(i) An imaginary closed surface is considered which passes through the point at which the electric intensity is to be evaluated. This closed surface is known as Guassian surface.

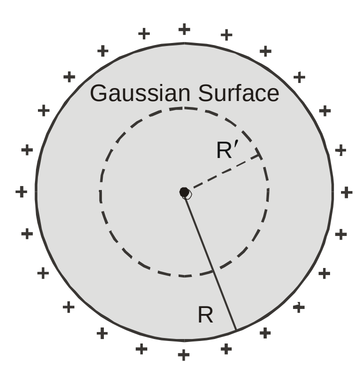
(ii) Charge enclosed by Guassian surface is calculated.

(iii) Electric flux through the Guassian surface is calculated.

(iv) Applying Gauss's law.

Q.14 Calculate the intensity of field inside a Hollow charged sphere.

Ans. **INTENSITY OF FIELD INSIDE A HOLLOW CHARGED SPHERE**



Consider a hollow conducting sphere of radius R is given a positive charge. Now imagine a sphere of radius $R' < R$ to be inscribed within the hollow charge sphere as shown in figure. The surface of this sphere is the Guassian surface. Let ϕ be the flux through this closed surface. It is clear from the figure that the charge enclosed by the Gaussian surface is zero.

i.e., $q = 0$

Applying Gauss's law

$$\phi_e = \frac{q}{\epsilon_o}$$

$$\phi_e = \frac{0}{\epsilon_o}$$

$$\phi_e = 0$$

Since from the definition:

$$\phi_e = \vec{E} \cdot \vec{A}$$

$$\therefore \quad \vec{E} \cdot \vec{A} = 0$$

As $\quad \vec{A} \neq 0$

Therefore, $\quad \vec{E} = 0$

Thus the interior of a hollow charged metal sphere is a field free region. As a consequence any apparatus placed within a metal enclosure is "shielded" from electric fields.

Q.15 Calculate the electric intensity due to an infinite sheet of charge.

Ans. **ELECTRIC INTENSITY DUE TO AN INFINITE SHEET OF CHARGE**

Suppose we have a plane sheet of infinite extent (size) on which positive charges are uniformly distributed. The uniform surface charge density is defined as charge per unit area.

Do You Know?

To eliminate stray electric field interference, circuits of sensitive electronic devices such as TV and Computers are often enclosed within metal boxes

i.e., $\quad \sigma = \dfrac{Q}{A}$

$$\therefore \quad Q = \sigma A \qquad \ldots\ldots \text{(i)}$$

where A is the surface area of the faces. A finite part of this sheet is shown in figure. To calculate electric intensity E at a point P, close to the sheet, imagine a closed Gaussian surface in the form of a cylinder passing through the sheet whose one flat face contains the point P. From symmetry we can conclude that $\vec{E}$ points at right angles to the end faces and away from the plane. Since $\vec{E}$ is parallel to the curved surface of the cylinder so there is no contribution to flux from the curved wall of the cylinder.

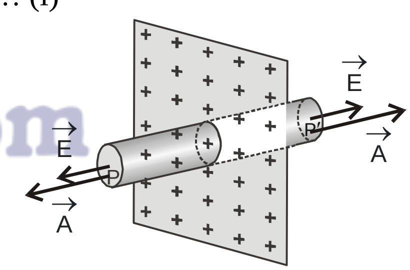
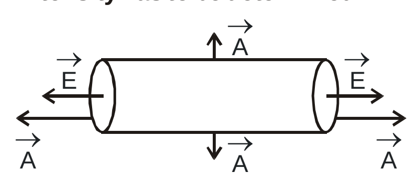


Fig. The closed surface is in the

contains the point P at which electric intensity has to be determined.



Now electric flux through a Gaussian surface is

$$\phi_e = \text{Flux through left face} + \text{Flux through right face} + \text{Flux through curved portion of cylinder}$$

$$\phi_e = EA\cos 0° + EA\cos 0° + EA\cos 90°$$

$$\phi_e = EA + EA + 0$$

$$\phi_e = 2EA \qquad \ldots\ldots \text{(ii)}$$

Applying Guass's law

$$\phi_e = \frac{1}{\epsilon_o} \times Q$$

Putting the values of Q and ϕ_e from eq. (i) and (ii)

$$2EA = \frac{1}{\epsilon_o}\sigma A$$

$$E = \frac{\sigma}{2\epsilon_o}$$

In vector form

$$\vec{E} = \frac{\sigma}{2\epsilon_o}\hat{r}$$

where $\hat{r}$ is a unit vector normal to the sheet directed away from it. If the sheet is negatively charged then,

$$\vec{E} = -\frac{\sigma}{2\epsilon_o}\hat{r}$$

Q.16 Calculate the electric intensity between two oppositely charged parallel plates.

Ans. **ELECTRIC INTENSITY BETWEEN TWO OPPOSITELY CHARGED PARALLEL PLATES**

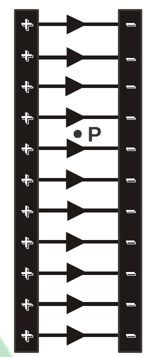


Fig. (1) The lines of force between the plates are normal to the plates and are directed from the positive plate towards the negative one.

Consider two parallel and closely spaced metal plates of infinite extent (size) separated by vacuum are given opposite charges. Under these conditions the charges are essentially concentrated on the inner surfaces of the plates. Thus the charges are uniformly distributed on the inner surface of the plates in a form of sheet of charges. Imagine now a Gaussian surface in the form of a hollow box with its top inside the upper metal plate and its bottom in the space between the plate as shown in Fig. (2).

The surface charge density

$$\sigma = \frac{Q}{A}$$

i.e., $Q = \sigma A$ …… (i)

where A is the area of the plate and Q is the amount of charge on either of the plates.

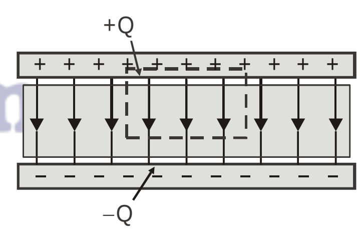


Fig. (2) Dotted rectangle represents the cross-section

inside the upper metal plate and its bottom in the dielectric between the plates.

Now the total flux through the Gaussian surface is

ϕ_e = Flux through front face + Flux through rear face
+ Flux through left face + Flux through right face
+ Flux through upper face + Flux through bottom face

os 90° ϕ_e =

+ EA cos 90° + 0 A cos θ

(there is no flux through the upper face of the box because there is no field inside the metal plate according to 1st application of Gauss's law) + EA cos 0°

$$\therefore \quad \phi_e = EA \qquad \left(\begin{array}{l}\therefore \ \cos 90^\circ = 0 \\ \ \ \ \cos 0^\circ \ = 1\end{array}\right)$$

Applying Gauss's law

$$\phi_e = \frac{1}{\epsilon_o}Q$$

Putting values of Q and ϕ_e

$$\therefore \quad EA = \frac{1}{\epsilon_o}\sigma A$$

$$E = \frac{\sigma}{\epsilon_o}$$

The field intensity is the same at all points between the plates. The direction of field from +ve to –ve plate because a unit positive charge anywhere between the plates would be repelled from +ve and attract to –ve plate so these forces are in the same direction.

In vector form

$$\vec{E} = \frac{\sigma}{\epsilon_o}\hat{r}$$

where $\hat{r}$ is a unit vector directed from positive to negative plate.

Q.17 Describe electric potential and potential difference with its units.

Ans.

ELECTRIC POTENTIAL

Consider a positive charge q_o which is allowed to move in an electric field produced between two oppositely charged parallel plates as shown in fig. (i). The positive charge will move from plate B to A and will gain K.E. If it is to be moved from A to B, an external force is needed to make the charge move against the electric field and will gain potential energy. As the charge is moved from A to B. It is moved keeping electrostatic equilibrium i.e., it moves with uniform velocity. This condition could be achieved by applying a force $\vec{F}$ equal and opposite to $\vec{E}q_o$ at every point along its path as shown in fig. (ii).

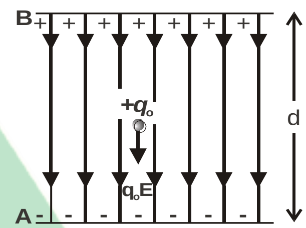


Fig. (i)

Electric Potential Energy

"The work done by external force against the electric field increases electric potential energy of the charge that is moved".

Let W_{AB} be the work done by the force in carrying the positive charge q_o from A to B while keeping the charge in equilibrium. This work is stored in the charge as **electric potential energy**.

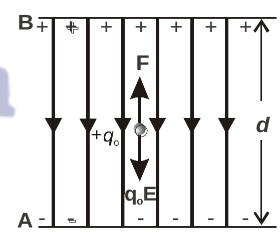


Fig. (ii)

The change in its P.E is

$$\Delta U = W_{AB}$$

$$U_B - U_A = W_{AB}$$

where U_A and U_B are defined to be the potential energies at A and B respectively.

Electric Potential Difference

The potential difference between two points A and B in an electric field is defined as

"The work done in carrying a unit positive charge from A to B against the electric field while keeping the charge in equilibrium.

i.e., $$\Delta V = V_B - V_A = \frac{W_{AB}}{q_o}$$

or $$\Delta V = \frac{\Delta U}{q_o} \qquad (\because W_{AB} = \Delta U) \qquad \ldots\ldots \text{(i)}$$

$$V_B - V_A = \frac{\Delta U}{q_o}$$

where V_A and V_B are electric potentials at A and B respectively.

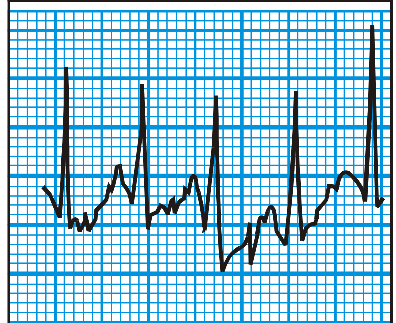
Electric P.E and electric P.D between the points A and B are related as

$$\Delta U = q_o \Delta V = W_{AB}$$

from eq. (i) **electric potential difference** can also be defined as

"The difference of potential energy per unit charge".

Do You Know?



An ECG records the "voltage" between points on human skin generated by electrical process in the heart. This ECG is made in running position providing information about the heart's performance under stress.

Unit of Potential Difference

The unit of P.D is J/C

As $$\Delta V = \frac{W_{AB}}{q_o} = \frac{J}{C}$$

It is also called volt. "A potential difference of 1 volt exist between two points if work done in moving a unit positive charge from one point to another keeping in equilibrium is 1 J".

Absolute Potential or Potential at a Point

The work done in bringing a unit positive charge from infinity to that point, (at which potential is to be determined) keeping it in equilibrium is called the absolute potential.

As $$V_B - V_A = \frac{W_{AB}}{q_o}$$

To calculate absolute potential at B we assume that point A is at infinity and take $V_A = 0$.

$$\therefore \quad V_B - 0 = \frac{W_{B\infty}}{q_o}$$

$$V_B = \frac{W_{B\infty}}{q_o}$$

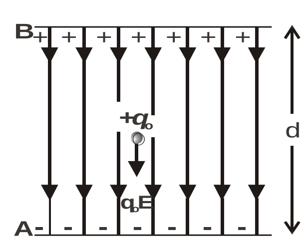
$$V = \frac{W}{q_o}$$

Potential at a point is potential difference between the potential at that point and potential at infinity. Both potential and potential difference are scalar quantities because both W (work) and q_o (charge) are scalars.

Q.18 *Explain the electric field as potential gradient. (OR) What is the relation between electric field and potential?*

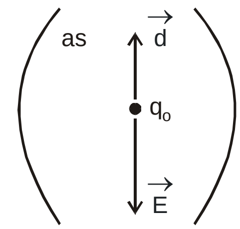
Ans. **ELECTRIC FIELD AS POTENTIAL GRADIENT**

The electric field E between the charged plates is uniform as shown in the figure. Let W_{AB} be the work done in moving charge q_o from plate A to B against the electric field. Now



$$\vec{F} = \vec{E}\, q_o$$

and $\vec{d}$ is the displacement covered by the charge then



$$\text{Work} = W = \vec{F} \cdot \vec{d}$$

$$\therefore \quad W_{AB} = q_o\, \vec{E} \cdot \vec{d}$$

$$W_{AB} = q_o\, Ed \cos 180^\circ \qquad (\cos 180^\circ = -1)$$

$$W_{AB} = q_o\, Ed\,(-1)$$

$$W_{AB} = -q_o\, Ed$$

From the definition of potential difference

$$\Delta V = \frac{W_{AB}}{q_o}$$

Putting value of W_{AB}

$$\therefore \quad \Delta V = -\frac{q_o\, Ed}{q_o}$$

$$\Delta V = -Ed$$

$$E = -\frac{\Delta V}{d}$$

If the plates A and B are separated by very small distance Δr then,

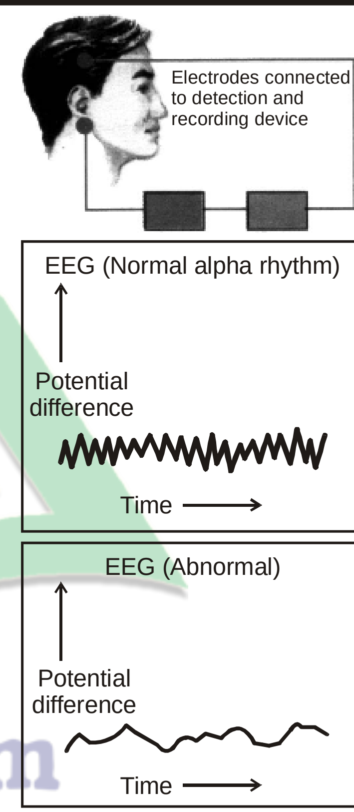
$$E = -\frac{\Delta V}{\Delta r}$$

The quantity $\frac{\Delta V}{\Delta r}$ gives the maximum value of rate of **"change of potential with distance"** because the charge has been moved along field lines along which the distance r between the two plates is minimum. It is known as potential gradient. Thus the electric intensity is equal to the negative of the gradient of potential. The negative sign indicates that **direction of E is along the decreasing potential**.

Unit

Another unit of electric intensity is volt/metre (V/m).

Do You Know?



In electroencephalography the potential differences created by the electrical activity of the brain are used for diagnosing abnormal behaviour.

Q. Show that V/m = N/C.

Ans. $\quad \text{L.H.S.} = \frac{V}{m}$

As $\quad V = \frac{J}{C}$

then $\quad \frac{V}{m} = \frac{J/C}{m} = \frac{J}{mC}$

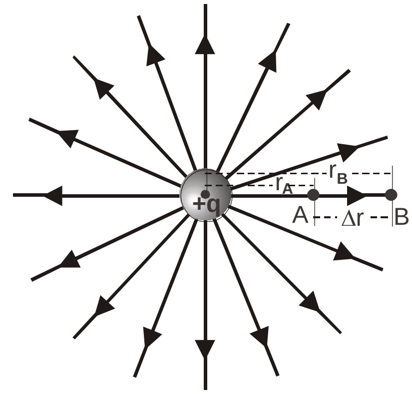
Since $\quad J = Nm$

then $\quad = \frac{Nm}{mC}$

$\quad = \frac{N}{C}$

Q.19 Define absolute potential. Also calculate the potential at a point due to a point charge.

Ans. **ELECTRIC POTENTIAL AT A POINT DUE TO A POINT CHARGE**



Consider a point charge q whose electric field intensity is $\vec{E}$. In order to calculate potential at point A we bring a unit positive charge from infinity to that point keeping the charge in equilibrium. This can be done by using $\Delta V = -\vec{E}\,\Delta r$ provided $\vec{E}$ remains constant. However in this case $\vec{E}$ varies as square of the distance from the point charge because $\left(E = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\right)$. To overcome this difficulty so we divide the field line into very small segments each of length Δr.

Consider two points A and B, very close to each other so that, $\vec{E}$ remains almost constant between them. The distance of points A and B from q are r_A and r_B respectively.

From figure

$$r_B = r_A + \Delta r$$

$$\Delta r = r_B - r_A$$

As r represents mid point of interval between A and B so

$$r = \frac{r_A + r_B}{2}$$

As the points A and B are very close then we can use a geometric average as

$$\frac{r}{r_A} = \frac{r_B}{r}$$

$$\therefore \quad r^2 = r_A r_B$$

Now if a unit positive charge is moved from B to A, the work done is equal to the potential difference between A and B

$$\therefore \quad \Delta V$$

$$V_A - V_B = -E(r_A - r_B) = E(r_B - r_A)$$

Do You Know?

Fish and other sea creatures produce electric fields in a variety of ways. Sharks have special organs, called the ampullae of Lorenzini, that are very sensitive to electric field and can detect potential difference of the order of

very precisely.

Putting the value of E

$$\therefore \quad V_A - V_B = \frac{1}{4\pi\in_o}\frac{q}{r^2}(r_B - r_A)$$

Putting the value of r^2

$$\therefore \quad V_A - V_B = \frac{q}{4\pi\in_o}\frac{1}{r_A r_B}(r_B - r_A)$$

$$V_A - V_B = \frac{q}{4\pi\in_o}\left(\frac{r_B}{r_A r_B} - \frac{r_A}{r_A r_B}\right)$$

$$= \frac{q}{4\pi\in_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$= \frac{1}{4\pi\in_o}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

To calculate absolute potential or potential at A, point B is supposed to be at infinity point so that $V_B = 0$ and $\frac{1}{r_B} = \frac{1}{\infty} = 0$.

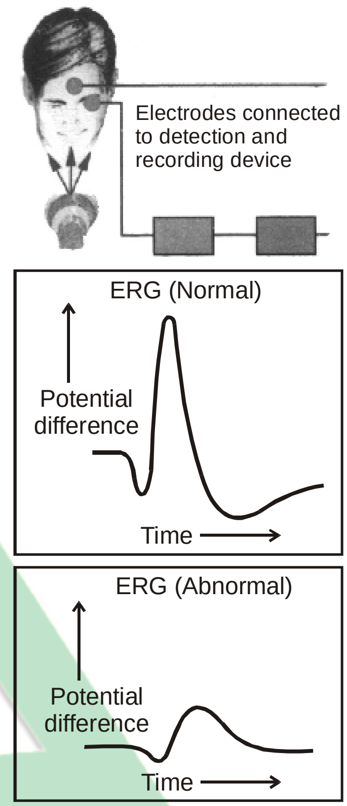
$$\therefore \quad V_A - 0 = \frac{q}{4\pi\in_o}\left(\frac{1}{r_A} - 0\right)$$

$$V_A = \frac{1}{4\pi\in_o}\frac{1}{r_A}$$

The general expression for electric potential V_r at a distance r from q is

$$V_r = \frac{1}{4\pi\in_o}\frac{1}{r}$$

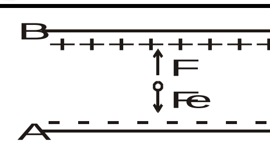
Do You Know?



The electrical activity of the retina of the eye generates the potential differences used in electroretinography.

Q.20 Define electron volt.

Ans. **ELECTRON VOLT**



We know when a charge q is moved from A to B keeping electrostatic equilibrium the change in potential energy is

$\Delta U = q$

If no external force acts on the charge to maintain equilibrium, this change in potential energy appears in the form of change in K.E. The energy gained by the charge will be

$$\Delta K.E = q\Delta V$$

$$= e\Delta V$$

$$= 1.6 \times 10^{-19}\ C \times \Delta V$$

Let $\quad \Delta V = 1$ volt

$$= 1.6 \times 10^{-19}\ C \times 1\ V$$

$$= 1.6 \times 10^{-19}\ C\ J/C \qquad (\because V = J/C)$$

$$= 1.6 \times 10^{-19}\ J$$

The amount of energy equal to 1.6×10^{-19} J is called 1 electron volt and is denoted by 1eV.

Definition of Electron Volt

"The amount of energy gained or lost by an electron as it moves through a potential difference of one volt is called electron volt (1 eV).

$$\therefore \quad 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

Note: Electron volt is the unit of energy in atomic physics.

Q.21 Compare the electric and gravitational forces.

Ans. COMPARISON OF ELECTRIC AND GRAVITATIONAL FORCES

Electric Force	Gravitational Force
(i) Electric force is a conservative force.	(i) It is also a conservative force.
(ii) It is inversely proportional to the square of distance i.e., $F = K\frac{q_1q_2}{r^2}$, $F \propto \frac{1}{r^2}$	(ii) It is also inversely proportional to the square of distance i.e., $F = G\frac{m_1m_2}{r^2}$, $F \propto \frac{1}{r^2}$
(iii) Electric force is a strong force.	(iii) Gravitational force is a weak force.
(iv) Electric force is attractive or repulsive force.	(iv) Gravitational force only a attractive force.
(v) Electric force depends upon the medium.	(v) It does not depend upon the medium.

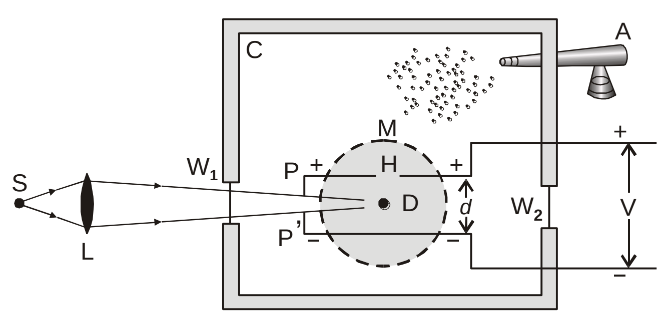
Q.22 Describe the charge on an electron by Millikan's method. (OR) How Millikan's method can be used to determine the charge?

Ans. CHARGE ON AN ELECTRON BY MILLIKAN'S METHOD

Introduction

In 1909, R.A Millikan devised a technique that resulted in precise measurement of charge on electron.

Construction



(i) Two parallel plates are placed in a container C to avoid disturbances due to air currents.

(ii) The upper plate P has a hole H in it.

(iii) The separation between the plates is d.

(iv) A voltage V is applied to the plates and an electric field is setup between the two plates.

(v) The magnitude of its value is $E = \frac{V}{d}$ (potential gradient).

(vi) An atomizer A is used for spraying oil drops into the container through nozzle. The oil droplets get charged due to the friction between walls of atomizer and oil droplets. These oil droplets are so small and are actually in term of mist.

(vii) The space between two plates is illuminated through source S through lens L and window W.

(viii) The path of motion of these drops can be carefully observed by a microscope M.

Working

Some of the these drops happen to pass through the hole. A given droplet between the two plates could be suspended in air if gravitational force Fg = mg acting on the droplet is equal to the electrical force Fe = qE.

$$\therefore \quad Fe = Fg$$

$$Eq = mg$$

$$q = \frac{mg}{E} \quad \text{...... (i)}$$

As we know $E = \frac{V}{d}$ so putting value of E in eq. (i)

$$\therefore \quad q = \frac{mg}{V/d}$$

$$q = \frac{mgd}{V} \quad \text{...... (A)}$$

Calculation of Mass 'm':

In order to determine mass m of the droplet, the electric field between the plates is switched off. The droplet fall under the action of gravity through air. It attains terminal speed v_t almost at the instant electric field is switched off. Its terminal speed v_t is determined by timing the fall of the droplet over a measured distance. Since drag force F due to air acting upon the droplet when it is falling with constant terminal speed is equal to its weight.

Hence by using Stoke's law

$$F_d = 6\pi\eta r v_t$$

$$F_d = mg$$

where r is the radius of the droplet and η is the coefficient of viscosity of air.

Comparing both

$$mg = 6\pi\eta r v_t \quad \text{...... (ii)}$$

$$m = \frac{6\pi\eta r vt}{g}$$

If ρ is the density of the droplet then

$$\rho = \frac{m}{v}$$

$$m = \rho v$$

$$m = \rho \frac{4}{3}\pi r^3 \qquad \ldots\ldots (B)$$

Putting value of m in eq. (ii)

$$\frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_t$$

$$\frac{4}{3}\rho r^2 g = 6\pi\eta v_t$$

$$r^2 = \frac{6\pi\eta v_t}{4/3\ \rho g}$$

$$r^2 = \frac{18\eta v_t}{4\rho g}$$

$$r^2 = \frac{18\eta v_t}{4\rho g}$$

$$r^2 = \frac{9\eta v_t}{2\rho g}$$

Knowing the value of r, the mass m can be calculated by using the above equation. The value of m is putting in equation (A) to get the value of charge q on droplet.

Millikan measured the charge on many droplet and found that each charge was an integral multiple of a minimum value of charge equal to 1.6×10^{-19} C. He therefore, concluded that this minimum value of the charge is the charge on an electron.

Q.23 Define capacitor and what is the parallel plate capacitor? Also define capacitance with its units.

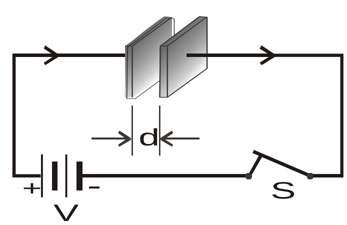
Ans. **CAPACITOR**

Definition

A capacitor is a device that can store charge.

Explanation

It consist of two conductors placed near one another separated by vacuum or air or any other insulator known as dielectric. As the conductors are in the form of parallel plates due to this the capacitor is known as **parallel plate capacitor**. When the plates of such a capacitor are connected to a battery of voltage V as shown in figure, it establishes a potential difference V volts between the two plates and the battery places a charge +Q on the plate connected with its positive terminal and a charge –Q on the other plate connected to its negative terminal.



For Your Information

One farad is an enormous amount of capacitance. For practical purposes its sub-multiple units are used which are given below:

1 micro-farad = 1 μF = 10^{-6} farad

1 pico-farad = 1 pF = 10^{-12} farad

Let Q is the magnitude of the charge on either of the plates. It is experimentally found that

$$Q \propto V$$

$$Q = CV$$

Where C is constant called the capacitance of the capacitor. It may be defined as the ability of a capacitor to store charge. Its value depends upon the geometry of the plates and the medium between them. It may be defined as the amount of charge on one plate necessary to raise the potential of that plate by one volt with respect to the other.

Mathematically

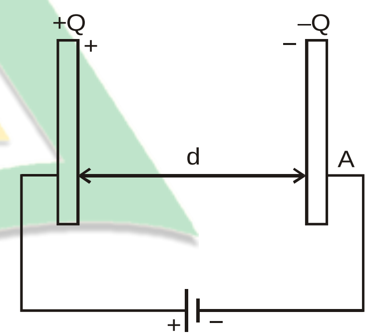
$$C = \frac{Q}{V} \qquad \ldots\ldots (*)$$

Unit of Capacitance

The SI unit of a capacitance is Farad. It may be defined as the capacitance of a capacitor is 1 Farad which stores one coulomb of charge (1C) when potential difference across the capacitor is one volt (1V).

Q.24 ***Describe the capacitance of a parallel plate capacitor. Also discuss the effect of medium on capacitance.***

Ans. CAPACITANCE OF A PARALLEL PLATE CAPACITOR



Consider a parallel plate capacitor consisting of two metal plates each of area A separated by distance d as shown in figure. The distance d is small so that electric field E between the plates is uniform. When the medium between the plates is air or vaccuum then

$$Q = CV$$

$$C_{vac} = \frac{Q}{V} \qquad \ldots\ldots (i)$$

where Q is the charge on capacitor and V is the potential difference between the plates.

The magnitude of electric intensity E is related with distance d by

$$E = \frac{V}{d} \qquad \ldots\ldots (ii) \qquad \left(E = -\frac{\Delta V}{\Delta r}\right)$$

As Q is the charge on either of the plates then surface density of charges on the plates is

$$\sigma = \frac{Q}{A}$$

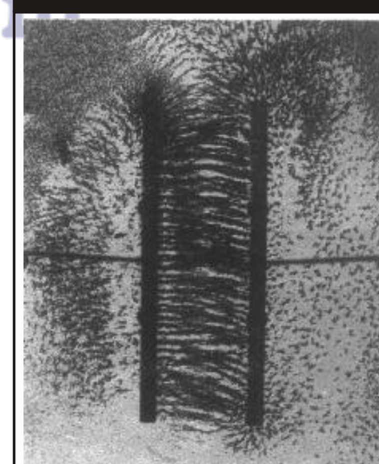
As we know electric intensity between two oppositely charged plates is

$$E = \frac{\sigma}{\epsilon_o} \qquad \text{(By 3}^{\text{rd}}\text{ application of Gauss's law)}$$

Putting the value of σ

$$\therefore \qquad E = \frac{Q}{A/\epsilon_o}$$

For Your Information



The electric field lines between the plates of a parallel-plate capacitor. Small bits of thread are suspended in oil and become aligned with the electric field. Note that the lines are equally spaced, indicating that the electric field there is uniform.

$$E = \frac{Q}{A\in_o} \qquad \text{...... (iii)}$$

From eq. (ii) and (iii)

$$\frac{V}{d} = \frac{Q}{A\in_o}$$

$$\frac{A\in_o}{d} = \frac{Q}{V}$$

As $\frac{Q}{V}$ is equal to C_{vac}

$$\therefore \qquad C_{vac} = \frac{A\in_o}{d} \qquad \text{...... (iv)}$$

This equation shows that the capacitance of a capacitor depends on area of the plates, distance between the plates and medium between them.

Effect of Medium on Capacitance

If an insulating material called dielectric of relative permitivity $\in_r$ is introduced between the plates, the capacitance of the capacitor enhanced by the factor $\in_r$. Capacitors commonly have some dielectric medium thereby $\in_r$ is also called dielectric constant. $\in_r$ is also called as dielectric constant.

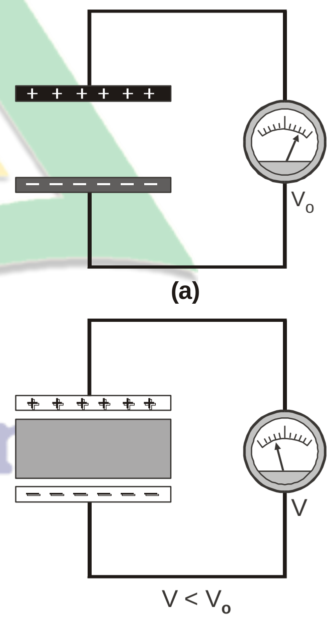


Fig. Effect of a dielectric on the capacitance of a capacitor.

For Your Information

A collection of capacitors used in various appliances.

Experiment

Consider a charged capacitor whose plates are connected to voltmeter as shown in figure. The deflection of the meter is a measure of potential difference between the plates. When a dielectric is inserted between the plates reading drops, indicating a decrease in the potential difference as shown in Fig. (b). From the definition

$$\text{As} \qquad C = \frac{Q}{V}$$

Since V decreases while Q remains constant, the value of C increases.

$$\therefore \qquad C_{med} = \frac{A\in_o\in_r}{d} \qquad \text{...... (v)}$$

Dividing eq. (v) by eq. (iv)

$$\frac{C_{med}}{C_{vac}} = \frac{A\in_o/d\ \in_r}{A\in_o/d}$$

$$\frac{C_{med}}{C_{vac}} = \in_r$$

Dielectric constant $\in_r$ is defined as

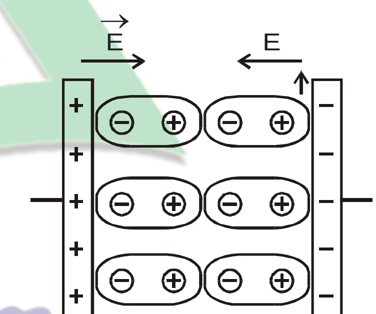
"The ratio of capacitance of a parallel plate capacitor with an insulating substance as medium between the plates to its capacitance with vacuum (or air) as medium between them".

Q.25 What is the electric polarization of dielectrics?

Ans. ELECTRIC POLARIZATION OF DIELECTRICS

The increase in the capacity of a capacitor due to presence of dielectric is due to electric polarization of dielectric.

The dielectric consists of atoms and molecules which are electrically neutral on the average, i.e., they contain equal amounts of negative and positive charges. The distribution of these charges in the atoms and molecules is such that the centre of the positive charge coincides with the centre of negative charge. When the molecules of dielectric are subjected to an electric field between the plates of a capacitor, the negative charges (electrons) are attracted towards the positively charged plate of the capacitor and the positive charges (nuclei) towards the negatively charged plate. The electrons in the dielectric (insulator) are not free to move but it is possible that the electrons and nuclei can undergo slight displacement when subjected to an electric field. As a result of this displacement the centre of positive and negative charges now no longer coincide with each other and one end of molecules shows a negative charge and the other end, an equal amount of positive charge but the molecule as a whole is still neutral. Two equal and opposite charges separated by a small distance are said to constitute a dipole. Thus the molecules of the dielectric under the action of electric field become dipoles and the dielectric is said to be polarized.



The effect of the polarization of dielectric is shown in figure. The positively charged plate attracts the negative end of the molecular dipoles and the negatively charged plate attracts the positive end. Thus the surface of the dielectric which is in contact with the positively charged plate places a layer of negative charges on the plate. Similarly the surface of the dielectric in contact with the negatively charged plate places a layer of positive charges. It effectively decreases the surface density of the charge σ on the plates. As the electric intensity E between the plates is $\frac{\sigma}{\varepsilon_o}$, so E decreases due to polarization of the dielectric. This results into a decrease of potential difference between the plates due to presence of dielectric.

Q.26 Calculate the energy stored in a capacitor.

Ans. ENERGY STORED IN A CAPACITOR

A capacitor is a device to store charge. It is also possible for a capacitor to store electrical energy. The charge on the plate possesses electrical potential energy which arises because work is to be done to deposit charge on the plates. Initial when the capacitor is uncharged the potential difference between the plates is zero and finally it becomes V when q charge is deposited on each plate. Thus the average potential difference is

$$\text{Average P.D} = \frac{0 + V}{2} = \frac{V}{2}$$

$$\therefore \text{ Electric potential energy} = q\,V_{ave}$$

$$= q \times \frac{V}{2}$$

$$= \frac{1}{2}qV$$

Since, $q = CV$

Electric potential energy $= \frac{1}{2}(CV)(V)$

$= \frac{1}{2}CV^2$

Energy stored $= \frac{1}{2}CV^2$ …… (i)

Energy in Terms of Electric Field

This energy stored in a capacitor in terms of electric field between the plates, so

$$V = Ed$$

and $$C = \frac{A\in_o\in_r}{d}$$

Putting in eq. (i)

$$\therefore \quad \text{Energy} = \frac{1}{2}\frac{A\in_o\in_r}{d}(Ed)^2$$

$$\text{Energy} = \frac{1}{2}\frac{A\in_o\in_r}{d}E^2d^2$$

$$\text{Energy} = \frac{1}{2}\in_o\in_rE^2(Ad)$$

which is the energy stored in the form of electric fields. As Ad is volume between the plates.

Energy Density

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}}$$

$$\text{Energy density} = \frac{1}{2}\frac{E^2\in_o\in_r(Ad)}{Ad}$$

$$\text{Energy density} = \frac{1}{2}E^2\in_o\in_r$$

This equation is valid for any electric field strength.

Q.27 Describe the charging and discharging of a capacitor.

Ans. **CHARGING AND DISCHARGING A CAPACITOR**

Many electric circuits consists of both capacitors and resistors.

Figure (i) shows a resistor-capacitor circuit called R.C circuit. When the switch is at terminal A, the R.C combination is connected to battery of voltage V_o which starts charging the capacitor through the resistor R.

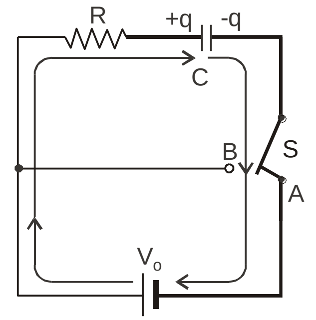


Fig. (i) Charging a capacitor

The capacitor is not charged immediately, rather charges build up gradually to the equilibrium value of $q_o = CV_o$. The growth of charge with time for different resistance is shown in figure (ii).

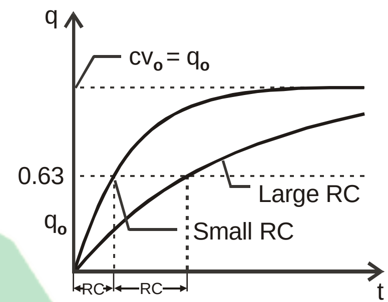


Fig. (Ii)

According to this graph q = 0 at t = 0 and increases gradually with time till it reaches its equilibrium value $q_o = CV_o$. The voltage V across capacitor at any instant can be obtained by dividing q by C as $V = q/C$.

The capacitor is charging or discharging, depends upon the product of resistance R and capacitance C used in circuit. As the unit of product of R.C is that of time, so this product is known as time constant.

In Fig. (iii) switch S is set at point B so the charge +Q on the left plate can flow anticlockwise through the resistance and neutralise the charge on the right plate. The graph (iv) shows that discharging begins at t = 0 when $q = CV_o$ and decreases gradually to zero. Smaller values of time constant, R.C lead to a more rapid discharge.

Important Points

- ♦ The resistance introduces the element of time in the charging and discharging of a capacitor.
- ♦ When a capacitor charges or discharges through a resistance, a certain time is required for the capacitor to charge fully or discharge fully. The voltage across a capacitor cannot charge instantaneously because. A finite time is requited to move charge from one point to another. The rate at which the capacitor charges or discharges is determined by the time constant of the circuit.

Time Constant

The time constant of a series R.C circuit is a time interval that equal to the product of the resistance and the capacitance. The time constant is express in seconds when resistance is in ohms and capacitance is in Farads.

$$t = RC$$

- As $I = \frac{Q}{t}$ the current depends on the amount of charge moved in a given time.
  - When the resistance is increased, the charging current is reduced, thus increasing the charging time of the capacitor.
  - When the capacitance is increased amount of charge increases, thus for the same current, more time is required to charge the capacitor.
- During time constant interval, the charge on a capacitor changes approximately 63%.

  An uncharged capacitor charges to 63% of its fully charged voltage in one time constant. When a capacitor is discharging its voltage drops approximately 37% of its initial value in one time constant which is a 63% change.
- It takes 5 times constant to approximately reach the final value.

  A five time constant interval is accepted as a time to fully charge or discharge a capacitor and is called the **transient time**.

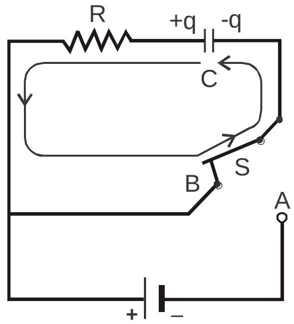


Fig. (iii) Discharging of a capacitor

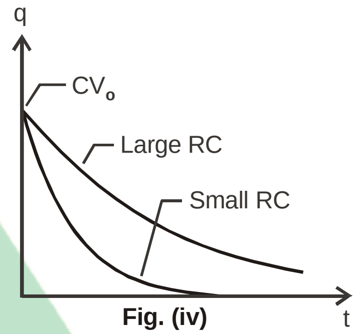


Fig. (iv)

Interesting Application

The charging/discharging of a capacitor enables some windshield wipers of cars to be used intermittently during a light drizzle. In this mode of operation the wipers remain off for a while and then turn on briefly. The timing of the on-off cycle is determined by the time constant of a resistor-capacitor combination.

SOLVED EXAMPLES

EXAMPLE 12.1

Charges q_1 = 100 μC and q_2 = 50 μC are located in xy-plane at positions $r_1 = 3.0\,\hat{j}$ and $r_2 = 4.0\,\hat{i}$ respectively, where the distances are measured in metres. Calculate the force on q_2.

Data

Charge = q_1 = 100 μC

= 100×10^{-6} C

Charge = q_2 = 50 μC

= 50×10^{-6} C

$\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$

Position vector = $\vec{r}_{21} = 4\hat{i} - 3\hat{j}$

To Find

Force on charge q_2 = $\vec{F}_{21}$ = ?

SOLUTION

By formula

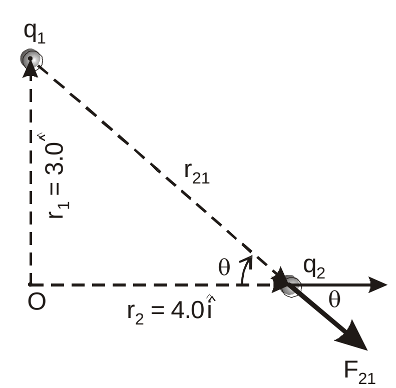
$$\vec{F}_{21} = K\,\frac{q_1 q_2}{r_2}\,\hat{r}_{21}$$

$$\hat{r}_{21} = \frac{\vec{r}}{r} = \frac{4\hat{i} - 3\hat{j}}{\sqrt{4^2 + 3^2}} = \frac{4\hat{i} - 3\hat{j}}{5}$$

So
$$\vec{F}_{21} = 9 \times 10^9 \times \frac{100 \times 10^{-6} \times 10 \times 10^{-6}}{52}\,\frac{(4\hat{i} - 3\hat{j})}{5}$$

$$= 1.44\,\hat{i} - 1.08\,\hat{j}$$

$$F_{21} = \sqrt{(1.44)^2 + (1.08)^2} = 1.8 \text{ N}$$



Result

Force on charge q_2 = F_{21} = 1.8 N

Direction of force = θ = –37° with x-axis

EXAMPLE 12.2

Two positive point charges $q_1 = 16.0\ \mu C$ and $q_2 = 4.0\ \mu C$ are separated by a distance of 3.0 m, as shown in figure. Find the spot on the line joining the two charges where electric field is zero.

Data

Charge $= q_1 = 16.0\ \mu C$

$= 16.0 \times 10^{-6}\ C$

Charge $= q_2 = 4.0\ \mu C$

$= 4.0 \times 10^{-6}\ C$

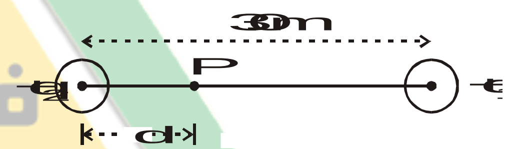
Distance between the charges $= r = 3.0\ m$

To Find

Distance where the field is zero $= d = ?$

SOLUTION

The electric intensity due to charge q_1 is



$$E_1 = \frac{1}{4\pi\epsilon_o}\ \frac{q_1}{(3-d)^2}$$

and the electric intensity due to charge q_2 is

$$E_2 = \frac{1}{4\pi\epsilon_o}\ \frac{r_2}{d^2}$$

At point P

$$E_1 = E_2$$

$$\frac{1}{4\pi\epsilon_o}\ \frac{q_1}{(3-d)^2} = \frac{1}{4\pi\epsilon_o}\ \frac{q_2}{d^2}$$

$$\frac{16.0 \times 10^{-6}}{(3-d)^2} = \frac{4.0 \times 10^{-6}}{d^2}$$

$$\sqrt{\frac{16.0}{(3-d)}} = \sqrt{\frac{4.0}{{}^{2}}}$$

$$\frac{4}{3-d} = \frac{2}{d}$$

$$4d = 2(3-d)$$

$$4d = 6 - 2d$$

$$4d + 2d = 6$$

$$6d = 6$$

$$d = \frac{6}{6}$$

$$d = 1$$

Result

Distance where field is zero $= d = 1.0\ m$

EXAMPLE 12.3

Two opposite point charges, each of magnitude q are separated by a distance 2d. What is the electric potential at a point P mid-way between them?

Data

Distance between the charges = 2d

Charge = +q

Charge = –q

To Find

Electric potential = V = ?

SOLUTION

As the potential due to +ve charge

$$V^{+} = \frac{1}{4\pi\epsilon_o}\frac{q}{d}$$

and the potential due to –ve charge

$$V^{-} = \frac{1}{4\pi\epsilon_o}\frac{-q}{d}$$

Since the total potential is

$$V = V^{+} + V^{-1}$$

$$= \frac{1}{4\pi\epsilon_o}\frac{q}{d} + \frac{1}{4\pi\epsilon_o}\frac{-q}{d}$$

$$V = 0$$

Result

Potential at a mid-point due to two opposite charges will be zero.

EXAMPLE 12.4

A particle carrying a charge of 2e falls through a potential difference of 3.0 V. Calculate the energy acquired by it.

Data

Magnitude of charge = q = 2e

Potential difference = V = 3.0 V

To Find

Energy acquired = E = ?

SOLUTION

The energy acquired by the particle is

$$\Delta K.E = q\Delta V$$
$$\Delta K.E = (2e)(3.0\ V)$$
$$\Delta K.E = 6\ eV$$
$$\Delta K.E = 6.0 \times 1.6 \times 10^{-19}\ J$$
$$\Delta K.E = 9.6 \times 10^{-19}\ J$$

Result

Energy acquired = $\Delta K.E$ = 9.6×10^{-19} J

EXAMPLE 12.5

In Millikan oil drop experiment, an oil drop of mass 4.9×10^{-15} kg is balanced and held stationary by the electric field between two parallel plates. If the potential difference between the plates is 750 V and the spacing between them is 5.0 mm, calculate the charge on the droplet. Assume $g = 9.8\ ms^{-2}$.

Data

Mass of droplet = m = 4.9×10^{-15} kg

Potential difference = V = 750 volt

Distance between plates = d = 5.0 mm

= 5.0×10^{-3} m

Value of g = $9.8\ m/s^2$

To Find

Charge on the droplet = q

SOLUTION

By formula

$$q = \frac{mgd}{V}$$
$$= \frac{4.9 \times 10^{-15} \times 9.8 \times 5.0 \times 10^{-3}}{750}$$
$$q = 3.2 \times 10^{-19}\ C$$

Result

Charge on the droplet = q = 3.2×10^{-19} C

EXAMPLE 12.6

The time constant of a series RC circuit is t = RC. Verify that an ohm times farad is equivalent to second.

SOLUTION

Ohm's law in terms of potential difference V, current I and resistance R can be written as

$$V = IR$$

Putting $I = \frac{q}{t}$

$$V = \frac{q}{t}R$$

or

$$R = \frac{V \times t}{q}$$

According to equation

$$q = CV \quad , \quad C = \frac{q}{V}$$

Multiplying this equation with above equation gives

$$RC = \frac{V \times t}{q} \times \frac{q}{V} = t$$

Hence 1 ohm × 1 farad = 1 second