30 Important Questions

1. Let f: [-10, 10] → R, where f(x) = sin x + [x2/a] be an odd function. Then set of values of parameter 'a' is/are:

(A) (-10, 10) - {0}

(B) (0, 10)   (C) [100,∞)

(D) (100,∞)

2. If f is a function such that f (0) = 2, f (1) = 3 and f(x+2) = 2f(x) – f(x+1) for every real x then f(5) is

(A) 7   (B) 13

(C) 1   (D) 5

3. The value of $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right)$ is

(A) π/2   (B) π/4

(C) π   (D) 2π

4. For a 3 × 3 matrix A, if det A = 4, then det (Adj A) equals

(A) –4   (B) 4

(C) 16   (D) 64

5. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is

(A) 3ω (B) 3ω(ω – 1)

(C) 3ω2 (D) 3ω(1 – ω)

6. The number of points at which the function f(x) = $\frac{1}{\log|x|}$ is discontinuous is

(A) 1 (B) 2

(C) 3 (D) 4

7. The set of all points where the function f(x) = x $|x|$ is differentiable is

(A) (– ∞, ∞) (B) (– ∞, 0) ∪ (0, ∞)

(C) (0, ∞) (D) [0, ∞]

8. If x is real, then maximum value of

y = 2(a – x) (x + $\sqrt{x^2 + b^2}$) is

(A) a2 + b2 (B) a2 – b2

(C) a2 + 2b2 (D) 2a2 + b2

9. The tangent at any point on the curve x3 + y3 = 2 cuts the intercepts p & q on the co–ordinate axis. The value of p–3/2 + q–3/2 equals

(A) √2 (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{2}}{5}$ (D) none of these

10. Let I1 = $\int_0^{3\pi} f(\cos^2 x)$ dx, $I_2 = \int_0^{2\pi} f(\cos^2 x)dx$ and $I_3 = \int_0^{\pi} f(\cos^2 x)$ dx, then

(A) I1 = 2I3 + 3I2 (B) I1 = 2I2 + I3

(C) I2 + I3 = I1 (D) I1 = 2I3

11. $\int \frac{\left(2+\sqrt{x}\right)}{\left(x+\sqrt{x}+1\right)^2}dx$ is equal to

(A) $\frac{2x}{x+\sqrt{x}+1}+c$

(B) $\frac{1}{x+\sqrt{x}+1}+c$

(C) $\frac{x}{x+\sqrt{x}+1}+c$

(D) $\frac{x}{x-\sqrt{x}+1}+c$

12. The area between the curve y = x (x –1) (x – 2) and x-axis is

(A) 1/4 (B) 1/2

(C) 1 (D) 0

13. Solution of $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)$, given that y = 1 when x = 1, is

(A) $\log\left|\frac{(x-y)^2-2}{2}\right| = 2(x+y)$

(B) $\log\left|\frac{(x-y)^2+2}{2}\right| = 2(x-y)$

(C) $\log\left|\frac{(x+y)^2+2}{2}\right| = 2(x-y)$

(D) None of these

14. The differential equation of all circles which pass through the origin and whose centres lie on y-axis is:

(A) $\left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$

(B) $\left(x^2-y^2\right)\frac{dy}{dx}+2xy=0$

(C) $\left(x^2-y^2\right)\frac{dy}{dx}-xy=0$

(D) $\left(x^2-y^2\right)\frac{dy}{dx}+xy=0$

15. Find the value of λ so that the points P, Q, R, S on the side OA, OB, OC and AB of a regular tetrahedron are coplanar. You are given that $\frac{\overrightarrow{OP}}{\overrightarrow{OA}} = \frac{1}{3}$; $\frac{\overrightarrow{OQ}}{\overrightarrow{OB}} = \frac{1}{2}$, $\frac{\overrightarrow{OR}}{\overrightarrow{OC}} = \frac{1}{3}$ and $\frac{\overrightarrow{OS}}{\overrightarrow{AB}} = \lambda$.

(A) λ = 1/2 (B) λ = –1

(C) λ = 0 (D) for no value of λ

16. The direction ratios of a normal to the plane passing through (1, 0, 0), (0, 1, 0) and making an angle $\frac{\pi}{4}$ with the plane x + y = 3 are

(A) $(1, \sqrt{2}, 1)$ (B) $(1, 1, \sqrt{2})$

(C) (1, 1, 2) (D) $(\sqrt{2}, 1, 1)$

17. If the plane $ax + by + cz = d$ intersects the co–ordinate axes at the points A, B and C, then the area of the triangle ABC is [a, b, c are d.c. of the normal to the plane]

(A) $\frac{d^2}{abc}$ (B) $\frac{d^2}{2abc}$

(C) $\frac{2d^2}{abc}$ (D) $\frac{abc}{d^2}$

18. In a sequence of independent trials, the probability of successes in one trial in $\frac{1}{4}$. Then the probability that second success takes place on or after the fourth trial, is

(A) $\frac{32}{37}$ (B) $\frac{22}{27}$

(C) $\frac{23}{27}$ (D) $\frac{27}{32}$

19. A person draws two balls successively without replacement from a box containing 10 red balls, 7 black balls and 5 green balls. He tells that both the balls are of green colour. What is the

probability that both are green balls if there are 75% chances that he speaks truth?

(A) $\frac{30}{251}$ (B) $\frac{60}{251}$

(C) $\frac{90}{251}$ (D) $\frac{20}{251}$

20. Maximise

z = 3x + 2y subject to the constraints:

$x - y \le 1$, $x + y \ge 3$; $x \ge 0$ and $y \ge 0$.

(A) 6

(B) 8

(C) 12

(D) Unbounded maximum

21. The number of words that can be written using all the letters of the word 'IRRATIONAL' is

(A) $\frac{10!}{(2!)^3}$

(B) $\frac{10!}{(2!)^2}$

(C) $\frac{10!}{2!}$

(D) 101

22. A series is given with one term missing. Select the correct alternative from the given ones that will complete the series.

AB, CD, EF, GH, ?

(A) HJ

(B) HK

(C) IJ

(D) JI

23. In the following question, select the odd number pair from the given alternatives.

(A) 9 – 90

(B) 6 – 42

(C) 5 – 30

(D) 4 – 36

24. A unit vector perpendicular to both $\hat{i}+\hat{j}+\hat{k}$ and $2\hat{i}+\hat{j}+3\hat{k}$ is

(A) $(2\hat{i}-\hat{j}-\hat{k})\sqrt{6}$

(B) $\frac{(2\hat{i}-\hat{j}-\hat{k})}{\sqrt{6}}$

(C) $2\hat{i}+\hat{j}+\hat{k}$

(D) $\frac{3\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

25. The function f(x) = $\frac{\ln(1+ax)-\ln(1-bx)}{x}$ is not defined at x = 0. The value which should be assigned to f at x = 0, so that it is continuous at x = 0, is:

(A) a – b

(B) a + b

(C) b – a
(D) None of these

26. The maximum value of |z| when the complex number z satisfies the condition $\left|z+\frac{2}{z}\right|=2$ is
(A) $\sqrt{3}$
(B) $\sqrt{3}+\sqrt{2}$
(C) $\sqrt{3}+1$
(D) $\sqrt{2}-1$

27. If $\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=3^{25}(x+iy),$ where x and y are real, then the ordered pair (x, y) is
(A) (–3, 0)
(B) (0, 3)
(C) (0, –3)
(D) $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

28. If $\frac{z-1}{z+1}$ is purely imaginary, then
(A) $|z|=\frac{1}{2}$
(B) |z| = 1
(C) |z| = 2
(D) |z| = 3

29. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only

one student passed in all the subjects. Then the number of students failing in all the three subjects

(A) is 12

(B) is 4

(C) is 2

(D) cannot be determined from given information.

30. A vehicle registration number consists of 2 letters of English alphabets followed by 4 digits, where the first digit is not zero. Then the total number of vehicles with distinct registration numbers is

(A) $26^2 \times 10^4$

(B) ${}^{26}P_2 \times {}^{10}P_4$

(C) ${}^{26}P_2 \times 9 \times {}^{10}P_3$

(D) $26^2 \times 9 \times 10^3$

MATHEMATICS
SOLUTIONS

1. (D)

Since f(x) is an odd function,

$\left[\frac{x^2}{a}\right] = 0$ for all $x \in [-10, 10]$

$\Rightarrow 0 \le \frac{x^2}{a} < 1$ for all $x \in [-10, 10] \Rightarrow a > 100$.

Hence (D) is the correct answer.

2. (B)

For x = 0, f(2) = 2f(0) – f(1) = 2 × 2 – 3 = 1,

for x = 1, f(3) = 2f(1) – f(2) = 6 – 1 = 5,

for x = 2, f(4) = 2f(2) – f(3) = 2 × 1 – 5 = -3,

for x = 3, f(5) = 2f(3) – f(4) = 2×5 – (-3)= 13.

Hence (B) is the correct answer.

3. (B)

$\because \tan^{-1}\left(\frac{1}{2r^2}\right) = \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right) = \tan^{-1}(2r+1) - \tan^{-1}(2r - 1)$

$$\Rightarrow \sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{2r^2}\right) = \tan^{-1}(3) - \tan^{-1}(1) + \tan^{-1}(5) - \tan^{-1}(3)$$

$$+ \ldots + \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$$

$= \tan^{-1}(2n+1) - \tan^{-1}1$

$$\Rightarrow \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right) = \tan^{-1}(\infty) - \tan^{-1}1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}.$$

4. (C)

Since A Adj A = | A | I

$$= \begin{bmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{bmatrix}$$

$\therefore$ det (A Adj A)

$$= \begin{vmatrix} |A| & 0 & 0 \\ 0 & |A| & 0 \\ 0 & 0 & |A| \end{vmatrix} = |A|^3$$

$\therefore$ $|A|\,|\text{Adj } A| = |A|^3$

$\therefore$ $|\text{Adj } A| = |A|^2 = (4)^2 = 16$

5. (B)

Given determinant = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

Applying, $R_1 \to R_1 + R_2 + R_3$, we get $\begin{vmatrix} 3 & 0 & 0 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$

$$= 3(-\omega - \omega^3 - \omega^4) = -3(2\omega + 1) = 3\omega(\omega - 1)$$

6. (C)

The function $\log |x|$ is not defined at x = 0, and hence x = 0 is a point of discontinuity. Also, for f(x) to be defined, $\log |x| \neq 0$ that is $x \neq \pm 1$.

Hence, 1 and –1 are also points of discontinuity.

Clearly f(x) is continuous for $x \in R - \{0, 1, -1\}$.

Thus, there are three points of discontinuity.

Hence (C) is the correct answer.

7. (A)

$f(x) = x\,|x| = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

$f'(x) = \begin{cases} 2x & \text{if } x > 0 \\ -2x & \text{if } x < 0 \end{cases}$

f(x) is differentiable for all $x \in R$ except possibly at x = 0.

But $f'(0^+) = f'(0^-) = 0$.

Hence f is differentiable everywhere.

Hence (A) is the correct answer.

8. (A)

Let $t = x + \sqrt{x^2 + b^2}$

$\Rightarrow \frac{1}{t} = \frac{1}{x + \sqrt{x^2 + b^2}} = \frac{\sqrt{x^2 + b^2} - x}{b^2}$

$\Rightarrow t - \frac{b^2}{t} = 2x$ and $t + \frac{b^2}{t} = 2\sqrt{x^2 + b^2}$

$\Rightarrow$ Thus $2(a - x)\left(x + \sqrt{x^2 + b^2}\right) = (2a - t + \frac{b^2}{t})\,(t)$

$\Rightarrow 2at - t^2 + b^2 = a^2 + b^2 - (a^2 - 2at + t^2)$

$\Rightarrow a^2 + b^2 - (a - t)^2$

$\Rightarrow y = 2(a - x)\,(x + \sqrt{x^2 + b^2}) \leq a^2 + b^2$

9. (B)

$x^3 + y^3 = 2$, Let any point on this curve is (x_1, y_1)

$3y^2 \frac{dy}{dx} = -3x^2 \quad \Rightarrow \frac{dy}{dx} = -\frac{x^2}{y^2} \quad \Rightarrow \left.\frac{dy}{dx}\right)_{(x_1, y_1)} = -\frac{x_1^2}{y_1^2}$

Equation of tangent at $(x_1, y_1) \Rightarrow y - y_1 = -\frac{x_1^2}{y_1^2}(x - x_1)$

$$\Rightarrow yy_1^2 + xx_1^2 = 2$$

Length of intercepts

$p = \frac{2}{y_1^2}, q = \frac{2}{x_1^2}$

How $p^{-3/2} + q^{-3/2} = \frac{2^{-3/2}}{y_1^{-3}} + \frac{2^{-3/2}}{x_1^{-3}} = 2^{-3/2}(x_1^3 + y_1^3) = 2^{-3/2}(2) =$

$2^{-1/2} = \frac{1}{\sqrt{2}}$

10. (C)

$I_1 = \int_0^{3\pi} f(\cos^2 x)\,dx$, $I_2 = \int_0^{2\pi} f(\cos^2 x)\,dx$, $I_3 = \int_0^{\pi} f(\cos^2 x)\,dx$

Period of $f(\cos^2 x)$ can be in the form of $\frac{\pi}{n}$ where $n \in N$

So $I_3 = \int_0^{\pi} f(\cos^2 x)\,dx$

$I_2 = \int_0^{2\pi} f(\cos^2 x)\,dx = 2\int_0^{\pi} f(\cos^2 x)\,dx = 2I_3$

$I_1 = \int_0^{3\pi} f(\cos^2 x)\,dx = 3\int_0^{\pi} f(\cos^2 x)\,dx = 3I_3$

So $I_1 = I_2 + I_3$

11. (A)

$$I = \int \frac{\left(\frac{2}{x^2} + \frac{1}{x\sqrt{x}}\right)dx}{\left(1 + \frac{1}{\sqrt{x}} + \frac{1}{x}\right)^2}$$

Put $1 + \frac{1}{\sqrt{x}} + \frac{1}{x} = t$

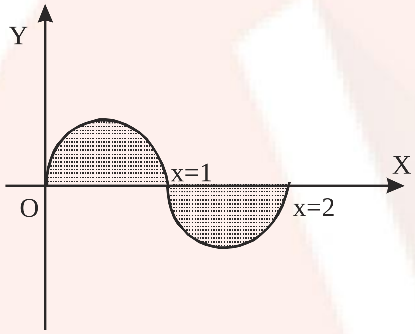
$$\Rightarrow -\left[\frac{1}{2x\sqrt{x}}+\frac{1}{x^2}\right]dx = dt$$

$$\Rightarrow \left[\frac{1}{x\sqrt{x}}+\frac{2}{x^2}\right]dx = -2dt$$

$$\therefore\ I = -2\int\frac{dt}{t^2} = \frac{2}{t}+c = \frac{2x}{x+\sqrt{x}+1}+c$$

12. (B)

Given curve meets x-axis at x = 0,1, 2



The required area is symmetrical about the point x = 1 as shown in the diagram. So

reqd. area = $2\int_0^1 y\,dx = 2\int_0^1 (x^3-3x^2+2x)\,dx$

$$= 2\left[\frac{x^4}{4}-x^3+x^2\right]_0^1 = 2\left(\frac{1}{4}-1+1\right) = \frac{1}{2}$$

13. (D)

$$\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \left(\frac{x+y+1}{x+y+2}\right)$$

Put x + y = t

$$1+\frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx}-1 = \left(\frac{t+1}{t+2}\right)\left(\frac{t-2}{t-1}\right)$$

$$\frac{dt}{dx} = \left(\frac{t^2 - t - 2}{t^2 + t - 2}\right) + 1$$

on solving we get

$$\Rightarrow 2(y - x) + \log\left(\frac{(x+y)^2 - 2}{2}\right) = 0$$

14. (A)

If (0, a) is centre on y-axis, then its radius is a because it passes through origin.

$\therefore$ Equation of circle is $x^2 + (y - a)^2 = a^2$

$\Rightarrow \quad x^2 + y^2 - 2ay = 0 \qquad ....(1)$

$\Rightarrow \quad 2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0 \qquad .....(2)$

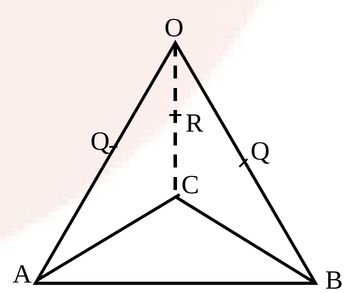
Using (1) in (2), $2x + 2y\frac{dy}{dx} - \frac{x^2 + y^2}{y}\frac{dy}{dx} = 0$

$$\Rightarrow 2xy = \left(x^2 + y^2 - 2y^2\right)\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

Hence (A) is the correct answer.

15. (B)



$\overrightarrow{OP} = \frac{\vec{a}}{3}, \ \overrightarrow{OQ} = \frac{\vec{b}}{2}, \ \overrightarrow{OR} = \frac{\vec{c}}{3}, \ \overrightarrow{OS} = \lambda(\vec{b} - \vec{a}),$ where $\vec{a}, \vec{b}, \vec{c}$ are the p.v. of the vertices *A*, *B*, *C*, '*O*' being the origin.

Equation of plane *PQR* is given as $\left(\vec{r} - \frac{\vec{a}}{3}\right).\vec{n} = 0$

where $\vec{n} = \left(\frac{\vec{c}}{3} - \frac{\vec{b}}{2}\right) \times \left(\frac{\vec{a}}{3} - \frac{\vec{b}}{2}\right) = \frac{\vec{c} \times \vec{a}}{9} - \frac{\vec{b} \times \vec{a}}{6} - \frac{\vec{c} \times \vec{b}}{6}$

Now since S lies on the plane $\Rightarrow \overrightarrow{OS}$ must satisfy equation of plane

$$\Rightarrow \left[\lambda(\vec{b} - \vec{a}) - \frac{\vec{a}}{3}\right] . \left(\frac{\vec{c} \times \vec{a}}{9} - \frac{\vec{b} \times \vec{a}}{6} - \frac{\vec{c} \times \vec{b}}{6}\right) = 0$$

$$\lambda \frac{b.c \times a}{9} + \lambda \frac{a.\vec{c} \times \vec{b}}{6} + \frac{a.\vec{c} \times \vec{b}}{18} = 0$$

$$\lambda \frac{[abc]}{9} - \frac{\lambda[abc]}{6} = \frac{[abc]}{18}$$

$\Rightarrow -3\lambda = 3 \qquad \Rightarrow \lambda = -1.$

16. (B)

Let the plane be $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\Rightarrow \frac{1}{a} = 1, \frac{1}{b} = 1 \Rightarrow a = b = 1.$

Also, $\cos\frac{\pi}{4} = \frac{\frac{1}{a} + \frac{1}{b}}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}\sqrt{1+1}} \Rightarrow c = \pm\frac{1}{\sqrt{2}}$

Thus direction ratios are $(1, 1, \sqrt{2})$ or $(1, 1, -\sqrt{2})$.

17. (B)

$A \equiv \left(\frac{d}{a}, 0, 0\right) \quad B \equiv \left(0, \frac{d}{b}, 0\right) \quad C \equiv \left(0, 0, \frac{d}{c}\right)$

$$A^2 = A_x^2 + A_y^2 + A_z^2 = \left(\frac{d^2}{2ab}\right)^2 + \left(\frac{d^2}{2bc}\right)^2 + \left(\frac{d^2}{2ca}\right)^2$$

or $A^2 = \frac{d^4}{4a^2b^2c^2}\left(c^2 + a^2 + b^2\right)$

$\therefore\ A = \dfrac{d^2}{2abc}$

As $\left[a^2 + b^2 + c^2 = 1\right]$.

18. (D)

The probability of second success taking place on fourth trial $= \left\{{}^3C_1 \times \frac{1}{4} \times \left(\frac{3}{4}\right)^2 \times \frac{1}{4}\right\}$ (i.e., one success on first three and one on fourth).

Similarly probability of second success taking place on fifth trial $= \left\{{}^4C_1 \times \frac{1}{4} \times \left(\frac{3}{4}\right)^3\right\} \times \frac{1}{4}$

$\therefore$ Required probability

$$= {}^3C_1\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2 + {}^4C_1\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 + {}^5C_1\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^4 + ...\infty$$

$$= \frac{9}{156}\left\{3 + 4\left(\frac{3}{4}\right) + 5.\left(\frac{3}{4}\right)^2 + ...\infty\right\}$$

$$= \frac{9}{256} \times 24 = \frac{27}{32} = \frac{27}{32}$$

19. (A)

E_1: both are green balls

E_2: both are not green balls

E: the person tells both are green balls

$$P(E) = P(E \cap E_1) + P(E \cap E_2)$$

$$= P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right)$$

$$= \left(\frac{5}{22}\times\frac{4}{21}\right)\left(\frac{3}{4}\right)+\left(1-\frac{5}{22}\times\frac{4}{21}\right)\left(\frac{1}{4}\right)$$

$$\therefore\ P\left(\frac{E_1}{E}\right)=\frac{P(E\cap E_1)}{P(E)}=\frac{30}{251}$$

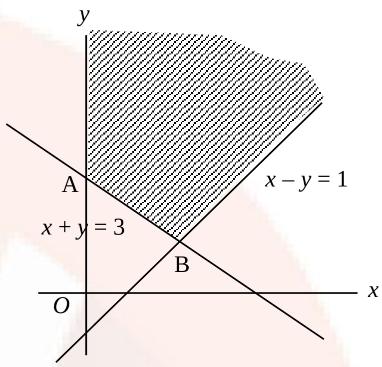
20. (D)

On solving A = (0, 3) and B = (2, 1)

The value of the objective function at these vertices are

z(A) = 6 and z(B) = 8.

But there exist points in the convex region for which the value of the objective function is more than 8. For instance, the point (10, 11) lies in the region and the function value at this point is 52 which is more than 8.



Hence the maximum value of z occurs at the point at infinity only and thus the problem has an unbounded solution.

21. (A)

In IRRATIONAL, word, there are 2 I, 2 R, 2 A, one T, one O, One N, One L. Total Number of words $= \frac{10!}{(2!)^3}$

22. (C)

English Alphabet series Next two letters after H.

23. (D)

Product of two consecutive numbers 9*10=90

Product of two consecutive numbers 6*7=42

Product of two consecutive number s5*6=30

Product of two non-consecutive numbers 4*9=36

24. (B)

A vector $\perp$ to the two given vectors (say $\vec{a}$ and $\vec{b}$) will be some scalar multiple of $\vec{a}\times\vec{b}$.

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 2 & 1 & 3\end{vmatrix}$

$=\hat{i}(3-1)+\hat{j}(2-3)+\hat{k}(1-2)=2\hat{i}-\hat{j}-\hat{k}$

But $\left|2\hat{i}-\hat{j}-\hat{k}\right|=\sqrt{6}$

Thus, $\frac{(2\hat{i}-\hat{j}-\hat{k})}{\sqrt{6}}$ is such a unit vector.

25. (B)

The function f(x) is continuous at x = 0, therefore the following condition must satisfy

LHL (at x = 0) = $\lim\limits_{x\to 0} f(x)$

$$=\lim_{x\to 0}\frac{\log(1+ax)-\log(1-bx)}{x}$$

$$=\lim_{x\to 0}\frac{\log(1+ax)}{x}-\lim_{x\to 0}\frac{\log(1-bx)}{x}$$

$$=\lim_{ax\to 0}\frac{\log(1+ax)}{ax}+b\lim_{bx\to 0}\frac{\log(1-bx)}{-bx}=a+b$$

Hence, f(0) = a + b

26. (C)

Given, $\left|z+\frac{2}{z}\right|=2$

We know that

$$\left|z+\frac{2}{z}\right|\ge |z|-\frac{2}{|z|}$$

$$\Rightarrow |z|-\frac{2}{|z|}\le 2$$

$$\Rightarrow |z|^2-2|z|-2\le 0$$

After solving, we get

$$\Rightarrow |z|\le \sqrt{3}+1$$

27. (D)

$$\left(\frac{3}{2}+i\frac{\sqrt{3}}{2}\right)^{50}=\left(\sqrt{3}\right)^{50}\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^{50}$$

$$=3^{25}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^{50}$$

$$=3^{25}\left(\cos\frac{50\pi}{6}+i\sin\frac{50\pi}{6}\right)$$

$$=3^{25}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$$

comparing with question, we get

$$x=\frac{1}{2},\ y=\frac{\sqrt{3}}{2}$$

28. (B)

Explanation:

Let $\frac{z-1}{z+1}=\lambda i$, such that $\lambda\in R$

On taking componendo and Dividendo

$$\frac{z-1+z+1}{z+1-z+1}=\frac{1+\lambda i}{1-\lambda i}$$

$z = \frac{1+\lambda i}{1-\lambda i}$; $|z| = \left|\frac{1+\lambda i}{1-\lambda i}\right| = 1$

29. (C)

Explanation:

$n(M \cup P \cup B) = n(M) + n(P) + n(B) – n(M \cap P) – n(PMB) – n(B \cap P) + n(M \cap P \cap B)$

given $n(M) = 50$, $n(P) = 45$, $n(B) = 40$;

$\Rightarrow$ $n(M\cap P) + n(P\cap B) + n(B\cap M) – 3(M\cap P\cap B) = 32$

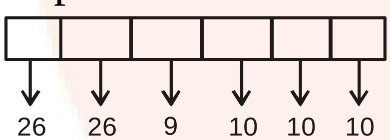
$99 = 50 + 45 + 40 – (32 + 3\, n(M \cap P \cap B) + n(M \cap P \cap B)$;

$2n(M \cap P \cap B) = 36 – 32$;

$n(M \cap P \cap B) = 2$

30. (D)

Explanation:



Total No. of ways = $26^2 \times 9 \times 10^3$