30 important questions

1. The sides of a triangle are 5, 12 and 13 units. A rectangle is constructed, which is equal in area to the triangle, and has a width of 10 units. Then the perimeter of the rectangle is

(A) 30 units (B) 36 units

(C) 13 units (D) None of these

Ans: D

Since 5-12-13 forms a Pythagorean triplet, the triangle under consideration is a right-angled triangle with height 12 and base 5. So area of the triangle $=\left(\frac{1}{2}\right)(12)(5)=30$ sq. units. If area of the rectangle with width 10 units is 30 sq. units, its length = 3 units. Hence, its perimeter = 2(10 + 3) = 26 units.

2. A stockist wants to make some profit by selling sugar. He contemplates about various methods. Which of the following would maximise his profit?

I. Sell sugar at 10% profit.

II. Use 900 g of weight instead of 1 kg.

III. Mix 10% impurities in sugar and selling sugar at cost price.

IV. Increase the price by 5% and reduce weights by 5%.

(A) I or III

(B) II

(C) II, III and IV

(D) Profits are same

Ans: B

Profit percentage in each case is

(i) 10%

(ii) $\frac{(100\times100)}{900} = \frac{100}{9}\%$

(iii) $\frac{100\times1.1.1-100}{100}\times100 = 10\%$

(iv) $\frac{(10\times100)}{95} = \frac{200}{19}\%$

3. A, B, C and D are four towns, any three of which are non-collinear. Then the number of ways to construct three roads each joining a pair of towns so that the roads do not form a triangle is

(A) 7 (B) 8

(C) 9 (D) 24

Ans: D

Let us choose a town, say A.

If I were to consider this as the base town and construct two roads such that I connect any pair of towns, I get the following combinations:

1. AB – BC,
2. AB – BD,
3. AC – CB,
4. AC – CD,
5. AD – DB and

6. AD – DC.

From any of these combinations, if I were to construct a road such that it again comes back to A, then it would form a triangle. To avoid a triangle, the third road that I construct should not be connected to A but to the third town.

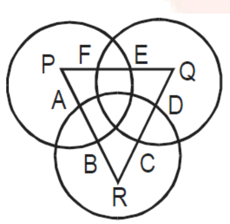
Hence, the combination would be:

1. AB – BC – CD,
2. AB – BD – DC,
3. AC – CB – BD,
4. AC – CD – DB,
5. AD – DB – BC and
6. AD – DC – CB.

Thus, from each town, we can construct 6 such combinations.

Hence, total number of combinations that we can have from four towns = (6 × 4) = 24.

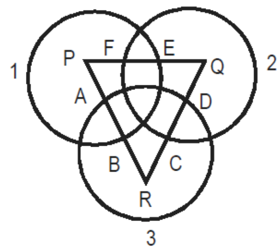
4. Three circles, each of radius 20, have centres at P, Q and R. Further, AB = 5, CD = 10 and EF = 12. What is the perimeter of ΔPQR?



(A) 120 (B) 66

(C) 93 (D) 87

Ans: C



PQ = PE + FQ – FE

= radius of circle 1+radius of circle 2–FE

= 20 + 20 – 12 = 28

Similarly, QR = 20 + 20 – CD = 40 – 10 = 30 and PR = 20 + 20 – AB = 40 – 5 = 35 So perimeter of ΔPQR=28+30+35 = 93

5. Distance between A and B is 72 km. Two men started walking from A and B at the same time towards each other. The person who started from A travelled uniformly with average speed of 4 km/hr. While the other man travelled with varying speed as follows: in the first hour his speed was 2 km/hr, in the second hour it was 2.5 km/hr, in the third hour it was 3 km/hr, and so on. When will they meet each other?

(A) 7 hr

(B) 10 hr

(C) 35 km from A

(D) Mid-way between A and B

Ans: D

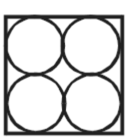
Since A and B are moving in opposite directions, we will add their speeds to calculate the effective speeds. In other words, in the first hour they would effectively cover a distance of (4 + 2) = 6 km towards each other. In the second hours, they would effectively cover a distance of (4 + 2.5) = 6.5 km towards each other.

In the third hour, (4 + 3) = 7 km.

In the fourth hour, (4 + 3.5) = 7.5 km and so on.

We can see that the distances that they cover in each hour are in AP, viz. 6, 6.5, 7, 7.5 ... with a = 6 and d = 0.5. Since they have to effectively cover a distance of 72 km, the time taken to cover this much distance would be the time taken to meet each other. So the sum of the first n terms of our AP has to be 72. If we are to express this in our equation of sum of first n terms of the AP, we will get $S_n = \frac{n}{2} \times [2a + (n-1)d]$ Substituting our values, we will get $72 = \frac{n}{2} \times [12 + 0.5(n-1)]$ Solving this, we get n = 9 hr. In 9 hr A would have covered (9 × 4) = 36 km. So B would also have covered (72 – 36) = 36 km. Hence, they would meet mid-way between A and B.

6. Four identical coins are placed in a square. For each coin the ratio of area to circumference is same as the ratio of circumference to area. Then find the area of the square that is not covered by the coins.



(A) 16(π – 1) (B) 16(8 – π)

(C) 16(4 – π) (D) $16\left(4 - \frac{\pi}{2}\right)$

Ans: C

Let R be the radius of each circle. Then $\frac{\pi R^2}{2\pi R} = \frac{2\pi R}{\pi R^2}$ which implies that $\frac{R}{2} = \frac{2}{R}$, i.e. $R^2 = 4$, i.e. R = 2.

Then the length of the square is 8. Thus, the area of the square is 64, while the area covered by each coin is $\pi 2^2 = 4\pi$. Since there are four coins, the area covered by coins is $4(4\pi) = 16\pi$. Hence, the area not covered by the coins is

$64 - 16\pi = 16(4 - \pi)$.

7. What is the volume of a cube whose surface area is twice that of a cube with volume 1?

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 8

Ans: C

Let x be the side length of the larger cube. The larger cube has surface area $6x^2$, and the smaller cube has surface area 6. So $6x^2 = 2 \cdot 6 = 12$, and $x = \sqrt{2}$. The volume of the larger cube is $x^3 = \left(\sqrt{2}\right)^3 = 2\sqrt{2}$

8. Older television screens have an aspect ratio of 4:3. That is, the ratio of the width to the height is 4:3. The aspect ratio of many movies is not 4:3, so they are sometimes shown on a television screen by \letterboxing"|darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of 2:1 and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?

(A) 2 (B) 2.25

(C) 2.5 (D) 2.7

Ans: D

9. Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let *t* be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by *t* ?

(A) $\left(\frac{1}{5}+\frac{1}{7}\right)(t+1)=1$

(B) $\left(\frac{1}{5}+\frac{1}{7}\right)t+1=1$

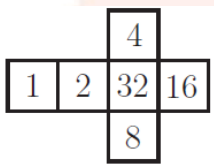
(C) $\left(\frac{1}{5}+\frac{1}{7}\right)t=1$

(D) $\left(\frac{1}{5}+\frac{1}{7}\right)(t-1)=1$

Ans: D

In one hour Doug can paint $\frac{1}{5}$ of the room, and Dave can paint $\frac{1}{7}$ of the room. Working together, they can paint $\frac{1}{5}+\frac{1}{7}$ of the room in one hour. It takes them *t* hours to do the job, but because they take an hour for lunch, they work for only *t*–1 hours. The fraction of the room that they paint in this time is $\left(\frac{1}{5}+\frac{1}{7}\right)(t-1)$ which must be equal to 1. It may be checked that the solution, $t=\frac{47}{12}$ does not satisfy the equation in any of the other answer choices.

10. Three cubes are each formed from the pattern shown. They are then stacked on a table one on top of another so that the 13 visible numbers have the greatest possible sum. What is that sum?



(A) 154 (B) 159

(C) 164 (D) 167

Ans: C

The sum of the six numbers on each cube is 1+2+4+8+16+32 = 63. The three pairs of opposite faces have numbers with sums 1 + 32 = 33, 2 + 16 = 18, and 4 + 8 = 12. On the two lower cubes, the numbers on the four visible faces have the greatest sum when the 4 and the 8 are hidden. On the top cube, the numbers on the five visible faces have the greatest sum when the 1 is hidden. Thus the greatest possible sum is

$3 \cdot 63 - 2 \cdot (4 + 8) - 1 = 164$.

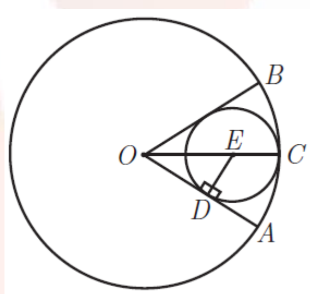
11. Points *A* and *B* lie on a circle centered at *O*, and $\angle AOB = 60^o$. A second circle is internally tangent to the first and tangent to both $\overline{OA}$ and $\overline{OB}$. What is the ratio of the area of the smaller circle to that of the larger circle?

(A) $\frac{1}{16}$ (B) $\frac{1}{9}$

(C) $\frac{1}{8}$ (D) $\frac{1}{6}$

Ans: B

Let r and R be the radii of the smaller and larger circles, respectively. Let E be the center of the smaller circle, let $\overline{OC}$ be the radius of the larger circle that contains E, and let Dbe the point of tangency of the smaller circle to $\overline{OA}$. Then $OE = R - r$, and because ΔEDOis a 30 – 60 { 90ºtriangle, OE = 2DE = 2r. Thus $2r = R - r$, so $\frac{r}{R} = \frac{1}{3}$. The ratio of the areas is $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$.



12. Let $k = 2008^2 + 2^{2008}$. What is the units digit of $k^2 + 2^k$?

(A) 0 (B) 2

(C) 4 (D) 6

Ans: D

The units digit of $2^n$is 2, 4, 8, and 6 for n = 1, 2, 3, and 4, respectively. For n >4, the units digit of $2^n$is equal to that of 2^{n-4}. Thus for every positive integer j the units digit of $2^{4j}$is 6, and hence 2^{2008} has a units digit of 6. The units digit of 2008^2 is 4. Therefore the units digit of k is 0, so the units digit of k^2 is also 0. Because 2008 is even, both 2008^2 and 2^{2008} are multiples of 4. Therefore k is a multiple of 4, so the units digit of 2^k is 6, and the units digit of $k^2 + 2^k$ is also 6.

13. A rectangular box has integer side lengths in the ratio 1 : 3 : 4. Which of the following could be the volume of the box?

(A) 48 (B) 56

(C) 64 (D) 96

Ans: D

Let the smallest side length be x. Then the volume is $x \cdot 3x \cdot 4x = 2x^3$. If x = 2 then $12x^3 = 96$

14. The mean, median, and mode of the 7 data values 60,100, x, 40, 50, 200, 90 are all equal to x. What is the value of x?

(A) 50 (B) 60

(C) 75 (D) 90

Ans: D

Since x is the mean,

$$x = \frac{60 + 100 + x + 40 + 50 + 200 + 90}{7}$$

$$x = \frac{540 + x}{7}$$

Therefore, 7x = 540 + x, so x = 90

15. How many ordered triples (x, y, z) of positive integers satisfy lcm(x, y) = 72, lcm (x, z) = 600 and lcm (y, z) = 900?

(A) 15 (B) 16

(C) 24 (D) 64

Ans: A

16. What is the value of $(2^0–1+5^2–0)^{–1}× 5$?

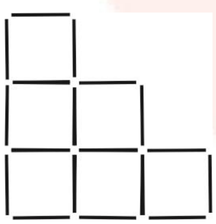
(A) – 125 (B) – 120

(C) $\frac{1}{5}$ (D) $\frac{5}{24}$

Ans: C

$(2^0 – 1 + 5^2 – 0)^{–1} × 5 = (1–1+25–0)^{–1}×5 = 25^{–1} × 5 = \frac{1}{25} \times 5 = \frac{1}{5}$

17. Ann made a 3-step staircase using 18 toothpicks as shown in the figure. How many toothpicks does the need to add complete a 5-step staircase?



(A) 9 (B) 22

(C) 18 (D) 24

Ans: B

We can see that a 1-step staircase requires 4 toothpicks and a 2-step staircase requires 40 toothpicks. Thus, to go from a 1-

setp staircase, 6 additional toothpicks are needed and to go from a
2-step to 3-step staircase, 8 additional toothpicks are needed, Applying this pattem, to go from a 3-step to 4-step staircase, 10 additional toothpicks are needed and to go from a 4-step to 5-step staircase, 12 additional toothpicks are needed. Our answer is 10 + 12 = 22.

18. Mr. Patrick teaches math to 15 Student. He was grading tests and found that when he graded everyone's test except Payton's, the average for the class was 80. After he graded Payton's test, the test average became 81. What was Payton's score on the test?

(A) 81 (B) 85

(C) 91 (D) 95

Ans: D

If the average of the first 14 peoples' scores was 80, then the sum of all off their tests is $14 * 80 = 1120$. When Payton's score was added, the sum of the all of the scores became $15 * 81 = 1215$. So, Payton's score must be $1205 – 1120 = 95$

19. The sum of two positive numbers is 5 times their difference. What is the ratio of the largest number to the smaller number?

(A) $\frac{5}{4}$ (B) $\frac{3}{2}$

(C) $\frac{9}{5}$ (D) $\frac{5}{2}$

Ans: C

Let a be the bigger number and b be the smaller.

a + b = 5(a – b)

Solving gives $\frac{a}{b} = \frac{3}{2}$, so the answer is $\frac{3}{2}$.

20. The ratio of the length of the width of a rectangle is 4 : 3. If the rectangle has diagonal o length d, then the area may be expressed as kd^2 for some constant k. What is k?

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$

(C) $\frac{12}{25}$ (D) $\frac{3}{4}$

Ans: C

Let the rectangle have length 4x and width 3x. Then by 3 – 4 – 5 triangles (or the Pythagorean Theorem), we have d = 5x, and so $x = \frac{d}{5}$. Hence, the area of the rectangle is $3x \cdot 4x = 12x^2 = \frac{12d^2}{25}$, so the answer is $\frac{12}{25}$.

21. Two years ago Pete was three times as old as his cousin Claire, Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be 2: 1?

(A) 2 (B) 4

(C) 6 (D) 8

Ans: B

This problem can be converted to system of equations. Let p be Pete's current age and c be Claire's current age.

The first statement can be written as $p - 2 = 3(c - 2)$. The second statement can be written as

$p - 4 = 4(c - 4)$

To solve the system of equations:

$p = 3c - 4$

$p = 4c - 12$

$3c - 4 = 4c - 12$

$c = 18$

$p = 20$

Let x be the number of years until Pete is twice as old as Claire.

$20 + x = 2(8 + x)$

$20 + x = 16 + 2x$

$x = 4$

22. Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(A) 0 (B) 1

(C) 2 (D) 3

Ans: B

23. Joey and his brothers are ages 3,5,7,9,11, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

(A) 3 (B) 7

(C) 9 (D) 11

Ans: D

Because the 5-year-old stayed home, we know that the 11-year-old did not go to movies, as the 5-year-old did not and 11+5=16. Also, the 11-year-old could not have gone to play baseball, as he is older than 10. Thus, the 11-year-old must have stayed home, so Joey is 11

24. In a recent basketball game, Shenille attempted only three-point shots and two-point shots, she was successful on 20% of her three-point shots and 30% of her two-point shote, Shenille attempted 30 shots. How many points did hse core?

(A) 12 (B) 18

(C) 24 (D) 36

Ans: B

Let the number of attempted three-point shots made be x and the number of attempted two-point shots be y. We know that $x + y = 30$, and we need to evaluate $(0.2 \cdot 3)x + (0.3 \cdot 2)y$, as we know that the three-point shots are both 3 points and that she made 20% of them and that the two-point shots are worth 2 and that she made 30% of them.

Simplifying, we see that this is equal to 0.6x + 0.6y = 0.6(x + y). Plugging in x + y = 30, we get 0.6(30) = 18

25. How many three-digit numbers are not divisible by 5, have digits that sum to less than 20, and have the first digit equal to the third digit?

(A) 52 (B) 60

(C) 68 (D) 70

Ans: B

26. A solid cube of side length 1 is removed from each corner of a solid cube of side length 3. How many edges does the remaining solid have?

(A) 36 (B) 60

(C) 84 (D) 108

Ans: C

We can use Euler's polyhedron formula that says that F + B = E + 2. We know that there are originally 6 faces on the cube, and each corner cube creates 3 more. 6 + 8(3) = 30. In addition each cube creates 7 new vertices while taking away the original 8, yielding 8(7) = 56 vertices. Thus E + 2 = 56 + 30, so E = 84.

27. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

(A) 6 (B) 9

(C) 13 (D) 16

Ans: C

We want the integers b such that 2013 ≡ 3 (mod b) ⇒ b is a factor of 2010. Since 2010 = 2 · 3 · 5 · 67, it has (1 + 1)(1 + 1)(1 + 1)(1 + 1) = 16 factors. Since b cannot equal 1,2, or 3, as these cannot have the digit 3 in their base representations, our answer is 16 – 3 = 13

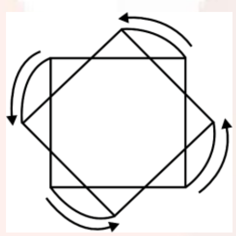
28. A unit square is rotated 45o about its center. What is the area of the region swept out by interior of the square?

(A) $1-\frac{\sqrt{2}}{2}+\frac{\pi}{4}$ (B) $\frac{1}{2}+\frac{\pi}{4}$

(C) $2-\sqrt{2}+\frac{\pi}{4}$ (D) $1+\frac{\sqrt{2}}{4}+\frac{\pi}{4}$

Ans: C

First, we need to see what this looks like. Below is a diagram.



For this square with side length 1, the distance from centre to vertex is $r=\frac{1}{2}$, hence the area is composed of a semicircle of radius r, plus 4 times a parallelogram with height $\frac{1}{2}$and base $\frac{\sqrt{2}}{2\left(1+\sqrt{2}\right)}$. That is to sky, the total area is

$\frac{1}{2}\pi\left(1/\sqrt{2}\right)^2+4\frac{\sqrt{2}}{4\left(1+\sqrt{2}\right)}=2-\sqrt{2}+\frac{\pi}{4}$.

29. Cagney can frost a cupcake every 20 second and lacey can frost a cupcake every 30 seconds. Working together, how many cupcakes can they frost in 5 minutes?

(A) 10 (B) 15

(C) 20 (D) 25

Ans: D

Cagney can frost one in 20 seconds, and Lacey can frost one in 30 seconds. Working together, they can frost one in $\frac{20 \cdot 30}{20+30} = \frac{600}{50} = 12$ seconds in 300 seconds (5 minutes), they can frost 25 cupcakes.

30. A square with side length 8 is cut in half, creating two congruent rectangles. What are the dimensions of one of these rectangles?

(A) 2 by 4 (B) 2 by 6

(C) 2 by 8 (D) 4 by 8

Ans: D

Cutting the square in half will bisect one pair of sides while the other side will remain unchanged. Thus, the new square is $\frac{8}{2} * 8$, or 4 by 8