PAPER 2

SECTION 01 LOGICAL REASONING

1. How many pairs of letters are there in the word "SACCHARINE" which have as many letters between them in the word as in the alphabet in both directions?
(A) 2
(B) 6
(C) 4
(D) 3
Ans. [D]
These are AC, AE and AC.

G M H N H V F Y T R O P H G S R Y W S Z S V C X J L K G

2. In the given series which of the following alphabet will be seventh to the left of ninth from the right alphabet?
(A) S
(B) T
(C) R
(D) H
Ans. [D]
It will be $7 + 9 = 16^{th}$ from right end that is H.

Directions (Q. 3 to 5) : *In each of the questions below statements are followed by four conclusions. Read the conditions and decide which logically follows from the given statements :*

3. **Statement :** (i) All doctors are fools.
(ii) All fools are illiterate.
Conclusions : I. All doctors are illiterate.
II. All illiterates are doctors.
III. All illiterates are fool.
IV. Some illiterates are doctors.
(A) Only I follows
(B) Only I and III follows
(C) Only I and IV follows
(D) II, III and IV follows

4. **Statements :** (i) Some alpha are beta.
(ii) All beta are gamma.
Conclusions : I. Some alpha are gamma.
II. No alpha is a gamma.
III. Some gamma are beta.
IV. No gamma is a beta.
(A) Only I follows

(B) II and III follows
(C) Only III follows
(D) I and III follows

Ans. [D]

5. **Statements :** (i) Some blankets are pillows.
(ii) All pillows are bananas.

Conclusions : I. Some bananas are blankets.
II. Some bananas are pillow.
III. No banana is a pillow.
IV. Some blankets are not bananas.

(A) I and III follows
(B) I and II follows
(C) I and IV follows
(D) Only I follows

Ans. [B]

Directions (Q. 6-8) : In each of the questions below, a group of numerals is given followed by four groups of symbol/letter combinations lettered (1), (2), (3) and (4). Numerals are to be coded as per the codes and conditions given below. You have to find out which of the combinations (1), (2), (3) and (4) is correct and indicate your answer accordingly.

Numerals: 3 5 7 4 2 6 8 1 0 9
Letter/Symbol Code: * B E A Ans. F K % R M
Following conditions apply:
(i) If the first digit as well as the last digit is odd, both are to be coded as 'X'.
(ii) If the first digit as well as the last digit is even, both are to be coded as '$'
(iii) If the last digit is '0', it is to be coded as '#'

6. 487692
(A) $KEFMAns.
(B) AKEFMAns.
(C) AKEFM$
(D) $KEFM$

Ans. [D]

7. 713540
(A) X%*BA#
(B) E%*BA#
(C) E%*BAR
(D) X%*BAR

Ans. [B]

8. 765082
(A) EFB#KAns.
(B) XFBRKAns.
(C) EFBRKAns.
(D) EFBR#K

Ans. [C]

9..Sunil remembers that his brother's birthday is after 15th but before 18th of February whereas his sister Kajol remembers that her brother's birthday is after 16th but before 19th of February. When is Sunil's brother's birthday?

(A) 16th

(B) 17th

(C) 15th

(D) 9th

Ans. [B]

According to Sunil, his brothers birthday fall on 16th or 17th February.

According to Kajol, Sunil's brother's birthday fall on 17th or 15th February.

So, common day is 17th February.

10. In a certain code, if STRONGER is written as TSSNOFFQ, then FREEDOM can be written as

(A) GFQRSNN

(B) GQFDENE

(C) GSFFEPN

(D) GQFDENN

Ans. [D]

S	T	R	O	N	G	E	R
+1	–1	+1	–1	+1	–1	+1	–1
T	S	S	N	O	F	F	Q
F	R	E	E	D	O	M	
+1	–1	+1	–1	+1	–1	+1	
G	Q	F	D	E	N	N	

11. Some boys are sitting in a row. P is sitting fourteenth from the left and Q is seventh from the right. If there are four boys between P and Q, then how many boys are there in the row?

(A) 19

(B) 21

(C) 25

(D) 28

Ans. [C]

14th 7th

|——— P ——— Q ———|

13 4 6

13 + 1 + 4 + 1 + 6

= 25

12. In a company, out of 80 workers, 24 can read English and 64 can read Telugu. How many workers can read both English and Telugu language?

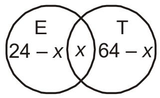
(A) 2

(B) 24

(C) 28

(D) 8

Ans. [D]



$24 - x + x + 64 - x = 80$
$88 - x = 80$
$x = 8$

Directions (Q. 13 to 15) : *Answer the following questions on the basis of the given conditions :*

(i) Village "P" is 70 km to the north of village "T".
(ii) Village "Q" is 30 km to the east of village "R" which is 30 km to the south of village "P".
(iii) Village "S" is 20 km to the west of village "T".
(iv) Village "U" is 15 km to the west of village "P".

13. Which village lies in south-west of P?
(A) S
(B) R
(C) Q
(D) T

Ans. [A]

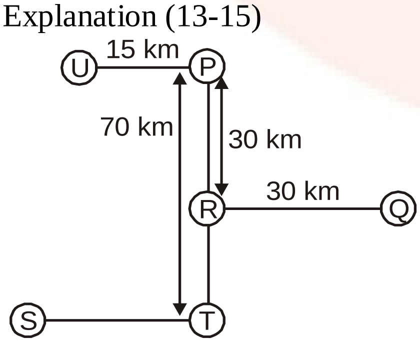
14. Which village is to the north-east of village "T"?
(A) S
(B) Q
(C) U
(D) T

Ans. [B]

15. Which village is farthest from village "P"?
(A) S
(B) U
(C) Q
(D) T

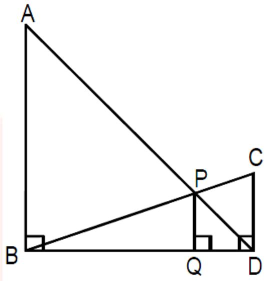
Ans. [A]

Explanation (13-15)



SECTION 02 MATHEMATICAL REASONING

16. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90^\circ$. If AB:CD = 3:1, the ratio of CD: PQ is



(A) 1 : 0.69

(B) 1 : 0.75

(C) 1 : 0.72

(D) None of the above

Ans. [B]

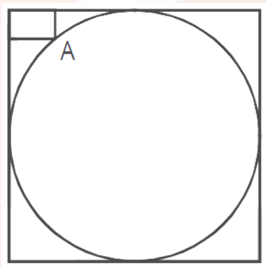
Using the Basic Proportionality Theorem, $\frac{AB}{PQ} = \frac{BD}{QD}$ and $\frac{PQ}{CD} = \frac{BQ}{BD}$

Multiplying the two we get,

$\frac{AB}{CD} = \frac{BQ}{QD} = 3:1$

Thus CD:PQ = BD:BQ = 4:3 = 1:0.75

17. In the figure below, the rectangle at the corner measures 10cm × 20cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



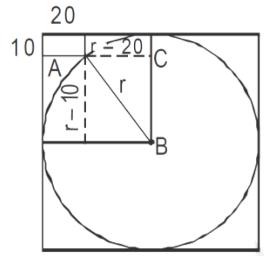
(A) 10 cm

(B) 40 cm

(C) 50 cm

(D) None of the above

Ans. [C]



Let the radius be r. Thus by Pythagoras' theorem for ΔABC we have $(r - 10)^2 + (r - 20)^2 = r^2$ i.e. $r^2 - 60r + 500 = 0$. Thus r = 10 or 50. It would be 10, if the corner of the rectangle had been lying on the inner circumference. But as per the given diagram, the radius of the circle should be 50 cm.

18. Let T be the set of integers {3, 11, 19, 27, …451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is

(A) 32 (B) 28

(C) 29 (D) 30

Ans. [D]

$T_n = a + (n - 1)d$

$467 = 3 + (n - 1)8$

$n = 59$

Half of n = 29 terms

29^{th} term is 227 and 30^{th} term is 235 and when these two terms are added the sum is less than 470.

Hence the maximum possible values the set S can have is 30.

19. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s, respectively. Then,

(A) $s > t > c$ (B) $c > t > s$

(C) $c > s > t$ (D) $s > c > t$

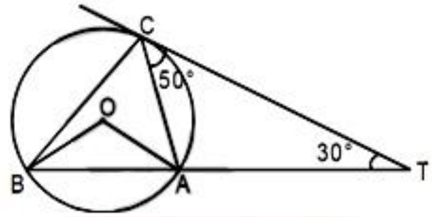
Ans. [C]

It's standard property among circle, square and triangle, for a given parameter, area of circle is the highest and area of the triangle is least whereas area of the square is in-between, i.e. $c > s > t$.

20. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O.

The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C.

If $\angle ATC = 30^o$ and $\angle ACT = 50^o$, then the angle $\angle BOA$ is



(A) 100^o

(B) 150^o

(C) 80^o

(D) Not possible to determine

Ans. [A]

21. Consider the sequence of numbers $a_1, a_2, a_3, ...$ to infinity where $a_1 = 81.33$ and $a_2 = -19$ and $a_j = a_{j-1} - a_{j-2}$ for $j \geq 3$. What is the sum of the first 6002 terms of this sequence?

(A) –100.33 (B) –30.00

(C) 62.33 (D) 119.33

Ans. [C]

Given $a_1 = 81.33$; $a_2 = -19$

Also:

$a_j = a_{j-1} - a_{j-2}$, for $j \geq 3$

$\Rightarrow a_3 = a_2 - a_1 = -100.33$

$a_4 = a_3 - a_2 = -81.33$

$a_5 = a_4 - a_3 = 19$

$a_6 = a_5 - a_4 = +100.33$

$a_7 = a_6 - a_5 = +81.33$

$a_8 = a_7 - a_6 = -19$

Clearly, a_7 onwards there is a cycle of 6 and the sum of terms in every such cycle = 0. Therefore, when we add $a_1, a_2, a_3...$ upto a_{6002}, we will eventually be left with $a_1 + a_2$ only i.e. $81.33 - 19 = 62.33$.

22. A shop stores x kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x?

(A) $2 \leq x \leq 6$ (B) $5 \leq x \leq 8$

(C) $9 \leq x \leq 12$ (D) $11 \leq x \leq 14$

Ans. [B]

23. Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer, then how many such triangles exist?

(A) 5 (B) 21

(C) 10 (D) 15

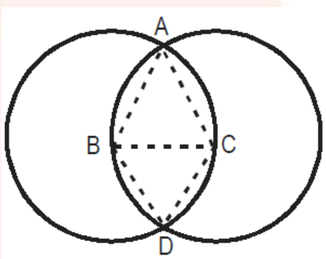
Ans. [C]

24. Two circles, both of radii 1 cm, intersect such that the circumference of each one passes through the centre of the other. What is the area (in sq. cm.) of the intersecting region?

(A) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ (B) $\frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

(C) $\frac{4\pi}{3} + \frac{\sqrt{3}}{2}$ (D) $\frac{2\pi}{3} - \frac{\sqrt{3}}{2}$

Ans. [D]



It is given that AB = BC = AC = BD = DC = 1 cm.

Therefore, ΔABC is an equilateral triangle.

Hence, $\angle ACB = 60°$

Now area of sector $\widehat{AB} = \frac{60}{360} \times \pi(1)^2 = \frac{\pi}{6}$

Area of equilateral triangle $\Delta ABC = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$

Area of remaining portion in the common region

ABC excluding ABC $= 2 \times \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

Hence, the total area of the intersecting region

$= 2 \times \frac{\sqrt{3}}{4} \times (1)^2 + 4 \times \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ sq.cm

25. Let $a_1 = p$ and $b_1 = q$, where p and q are positive quantities. Define

$a_n = pb_{n-1}$, $b_n = qb_{n-1}$, for even $n > 1$,

and $a_n = pa_{n-1}$, $b_n = qa_{n-1}$, for odd $n > 1$.

Which of the following best describes $a_n + b_n$ for even 'n'?

(A) $q(pq)^{\frac{1}{2}n-1}(p+q)$ (B) $pq^{\frac{1}{2}n-1}(p+q)$

(C) $q^{\frac{1}{2}n}(p+q)$ (D) $q^{\frac{1}{2}n}(p+q)^{\frac{1}{2}n}$

Ans. [A]

$a_n + b_n$ (n is even) $= p^{\frac{n}{2}}q^{\frac{n}{2}} + p^{\frac{n}{2}-1}q^{\frac{n}{2}+1}$

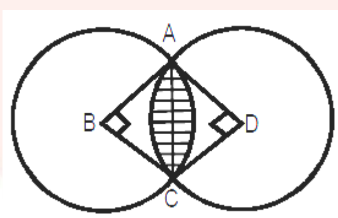
$= q(pq)^{\frac{n-1}{2}}(p+q)$

26. Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. The area in sq. cm of the portion that is common to the two circles is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2} - 1$

(C) $\frac{\pi}{5}$ (D) $\sqrt{2} - 1$

Ans. [B]



Shaded area = 2×(area of sector ADC – area of ΔADC)

$= 2 \times \left(\frac{\pi}{4} \times 1^2 - \frac{1}{2} \times 1 \times 1\right) = \frac{\pi}{2} - 1$

Hence option (B)

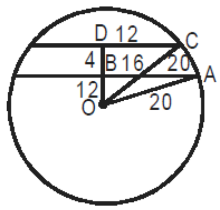
27. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?

(A) 1 or 7 (B) 2 or 14

(C) 3 or 21 (D) 4 or 28

Ans. [D]

Case I:Chords on same side of the centre.



OB2 = OA2 – AB2 = 202 – 162 = 144

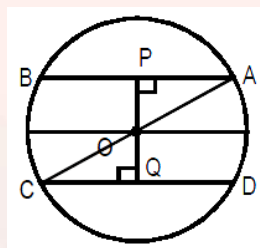
OB = 12

OD2 = 202 – 122 = 400 – 144 = 256

OD = 16

BD = 4 cm

Case II: Chords on opposite side of the centre.



AB = 32 cm

CD = 24 cm

$OP = \sqrt{AO^2 - AP^2} = \sqrt{(20)^2 - (16)^2}$

OP = 12 cm

& $OQ = \sqrt{(OC)^2 - (CQ)^2} = \sqrt{(20)^2 - (12)^2}$

OQ = 16 cm

Distance = PQ = 12 + 16 = 28 cm.

28. The rightmost non-zero digits of the number $30^{2720}$is

(A) 1 (2) 3

(C) 7 (D) 9

Ans. [A]

$((30)^4)^{680} = (8100)^{680}$.

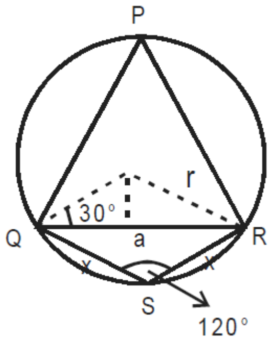
Hence, the right most non-zero digit is 1.

29. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral PQSR?

(A) $2r\left(1+\sqrt{3}\right)$ (B) $2r\left(2+\sqrt{3}\right)$

(C) $r\left(1+\sqrt{5}\right)$ (D) $2r+\sqrt{3}$

Ans. [A]



Here $\cos 30^\circ = \frac{a}{2r}$

$a = r\sqrt{3}$

Here the side of equilateral triangle is $r\sqrt{3}$

From the diagram $\cos 120^\circ = \frac{x^2 + x^2 - a^2}{2x^2}$

$a^2 = 3x^2$

$x = r$

Hence the circumference will be $2r(1+\sqrt{3})$

Hence answer is (A).

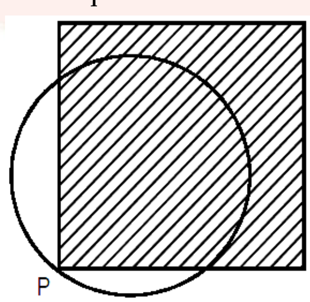
30. Let $x = +\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-....\text{to infintity}}}}}$. Then x equals

(A) 3 (B) $\left(\frac{\sqrt{13}-1}{2}\right)$

(C) $\left(\frac{\sqrt{13}+1}{2}\right)$ (D) $\sqrt{13}$

Directions for questions 31 and 32: Answer questions on the basis of the information given below:

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square.



Ans. [C]

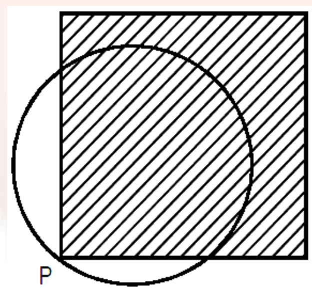
$x = \sqrt{4+\sqrt{4-x}} \Rightarrow x^2 = 4+\sqrt{4-x}$

$\Rightarrow \left(x^2-4\right) = \sqrt{4-x}$

Now putting the values from options, we find only option (3) satisfies the condition.

Directions for questions 50 and 51: Answer questions on the basis of the information given below:

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square.



31. The proportion of the sheet area that remains after punching is:

(A) $\frac{(\pi+2)}{8}$ (B) $\frac{(6-\pi)}{8}$

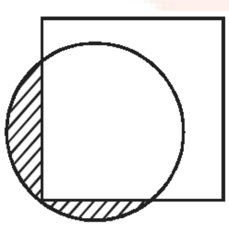
(C) $\frac{(4-\pi)}{4}$ (D) $\frac{(\pi-2)}{4}$

32. Find the area of the part of the circle (round punch) falling outside the square sheet.

(A) $\frac{\pi}{4}$ (B) $\frac{(\pi-1)}{2}$

(C) $\frac{(\pi-1)}{4}$ (D) $\frac{(\pi-2)}{2}$

Ans. [D]



$\text{Area} = \pi(1)^2 - \left(\frac{\pi}{2}+1\right) = \pi - \frac{\pi}{2} - 1 = \frac{\pi-2}{2}$

33. The number of solutions of the equation 2x + y = 40 where both x and y are positive integers and x ≤ y is:

(A) 7 (B) 13

(C) 14 (4) 18

Ans. [B]

$2x + y = 40$

$x \le y$

$\Rightarrow y = 40 - 2x$

Values of x and y that satisfy the equation

x	y
1	38
2	36
3	34
.	.
.	.
.	.
13	14

∴ 13 values of (x, y) satisfy the equation such that $x \le y$

34. The average of nine consecutive natural numbers is 81. Find the largest of these numbers.

(A) 80 (B) 85

(C) 78 (4) 94

Ans. [B]

Let numbers be x. x+1, x+2, x+3, x+4, x+5,x+6,x+7,x+8

Average=81 => 9x+36/9=81

=> x+4=81 => x=77

Hence Largest number = 85

35. Find the largest prime factor of 314 + 313 – 12 ?

(A) 73 (B) 81

(C) 69 (4) 54

Ans. [A]

$3^{14}+3^{13}-12$

$= 3^{13}(3+1) - 12 = 3.4(3^{12}-1)$

$= 3.4(3^6-1)(3^6+1)$

$= 3.4.(3^2-1)(3^4+3^2+1)(3^2+1)(3^4-3^2+1)$ $= 3.4.8.91.10.73 = 2^6.3.5.7.13.73$

Largest prime factor of $3^{14}+3^{13}-12 = 73$

SECTION 02 EVERY DAY MATHEMATICS

36. Find the number of perfect cubes between 1 and 1000001 which are exactly divisible by 7?

(A) 7 (B) 13

(C) 14 (4) 18

Ans. [C]

Number of perfect cubes between 1 and 1000001, which are exactly divisible by 7 must be cubes of numbers between 1 and 100 that are exactly divisible by 7.

Therefore, required number of such cubes = 14

37. Find the number of perfect cubes between 1 and 1000009 which are exactly divisible by 9.

(A) 33 (B) 37

(C) 47 (4) 39

Ans. [A]

Perfect cubes divisible by 9 will be cubes of multiples of 3.

Since, $1 < x^3 < 1000009$

ie. $1 < x < 101$

Also x is a multiple of 3

But, 101 = 3 x 33 + 2

Between 1 and 101 there are 33 multiples of 3

Required number of perfect cubes = 33

38. Find the remainder when 2^{2005} is divided by 13

(A) 21 (B) 13

(C) 14 (4) 19

Ans. [B]

$2^{2005}=2^{2000}.2^5$

$2^5=6(\text{mod } 13)$

$2^{10}= (25)^2 =6^2(\text{mod}13) =10(\text{mod}13)$

$2^{20}= (2^{10})^2 =10^2(\text{mod}13)= 9(\text{mod}13)$

$2^{40}= (2^{20})^2 =9^2(\text{mod}13) =3(\text{mod}13)$

$2^{200}= (2^{40})^5 =3^5(\text{mod}13)= 9(\text{mod}13)$

$2^{400}=(2^{200})^2 =9^2(\text{mod}13)= 3(\text{mod}13)$

$2^{2000}=(2^{400})^5 =3^5(\text{mod}13) =9(\text{mod}13)$

$2^{2005}=2^{2000}.2^5 =6.9\ (\text{mod}13)= 2(\text{mod}13)$

Remainder is 2 when 22005 is divided by 13

39. A triangular array of 2016 coins has 1 coin in the first row, 2 coins in the second row, 3 coins in the third row and so on up to N coins in the Nth row, What is the sum of the digits of N?

(A) 6 (B) 7

(C) 8 (D) 9

Ans. [D]

We are trying to find the value of N such that

1+2+3 ....+(N–1)+N $= \frac{N(N+1)}{2} = 2016$

Noticing that $\frac{63 \cdot 64}{2} = 2016$, we have n = 63, so our answer is 9.

Notice that we were attempting to solve $\frac{N(N+1)}{2} = 2016 \Rightarrow N(N+1) = 2016 \cdot 2 = 4032$

Approximating $N(N + 1) \approx N^2$, we were looking for a square that is close to, but less than, 4032 Since $64^2 = 4096$, we see that N = 63 is a likely candidate multiplying $63 \cdot 64$ confirms that our assumption is

40. For some positive integer n, the $110n^3$ has 110 positive integer divisors, including 1 and the number $110n^3$. How many positive integer divisors does the number $81n^4$ have?

(A) 110 (B) 191

(C) 325 (D) 425

Ans. [C]

Since the prime factorization of 10 is $2 \cdot 5 \cdot 11$, we have that the number is equal to $2 \cdot 5 \cdot 11 \cdot n^3$. This has $2 \cdot 2 \cdot 2 = 8$ factors when n = 1. This needs a multiple of 11 factors, which we can which we can achieve by setting $n = 2^3$, so we have $2^{10} \cdot 5, 11$ has 44 factors. To achieve the desired 110 factors we need the number of factors to also be divisible by 5, so we can set $n = 2^3 \cdot 5$, so has 110 factors. Therefore, $n = 2^3 \cdot 5$. In order to find the number of factors of $81n^4$, we raise this to the fourth power and multiply it by 81, and find the factors of that number. We have $3^4 \cdot 2^{12} \cdot 5^4$, and this has $5 \cdot 13 \cdot 5 = 325$ factors.

41. How many five-digit numbers can be formed using the digits 2, 3, 8, 7, 5 exactly once such that the number is divisible by 125?

(A) 0 (B) 1

(C) 4 (D) 3

Ans. [C]

Let us find some of the smaller multiples of 125. They are 125, 250, 375, 500, 625, 750, 875, 1000 ...A five-digit number is divisible by 125, if the last three digits are divisible by 125. So the possibilities are 375 and 875, 5 should come in unit's place, and 7 should come in ten's place. Thousand's place should contain 3 or 8. We can do it in 2! ways. Remaining first two digits, we can arrange in 2! ways. So we can have $2! \times 2! = 4$ such numbers. There are: 23875, 32875, 28375, 82375.

Direction for questions 42 and 43: Answer the questions based on the following information. In a locality, there are five small cities: A, B, C, D and E. The distances of these cities from each other are as follows.

AB = 2 km	AC = 2km
AD > 2 km	AE > 3 km
BC = 2 km	BD = 4 km
BE = 3 km	CD = 2 km
CE = 3 km	DE > 3 km

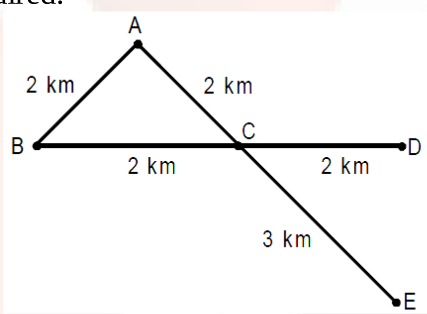
42. If a ration shop is to be set up within 2 km of each city, how many ration shops will be required?

(A) 2 (B) 3

(C) 4 (D) 5

Ans. [A]

If there is a shop at C, all A, B, C and D are within 2 km range. Another shop is needed for E. Hence, 2 shops are required.



43. If a ration shop is to be set up within 3 km of each city, how many ration shops will be required?

(A) 1 (B) 2

(C) 3 (D) 4

Ans. [A]

If there is a shop at C; all A, B, D and E are within 3 km range. Hence, 1 shop is required.

44. If n is any odd number greater than 1, then $n(n^2 - 1)$ is

(A) Divisible by 96 always

(B) Divisible by 48 always

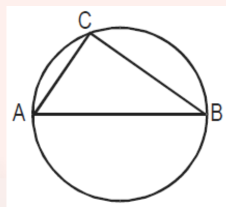
(C) Divisible by 24 always

(D) None of these

Ans. [C]

$n(n^2 - 1) = (n - 1)n(n + 1)$. If you observe, this is the product of three consecutive integers with middle one being an odd integer. Since there are two consecutive even numbers, one of them will be a multiple of 4 and the other one will be multiple of 2. Hence, the product will be a multiple of 8. Also since they are three consecutive integers, one of them will definitely be a multiple of 3. Hence, this product will always be divisible by $(3 \times 8) = 24$.

45. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm long, find the area of ΔABC.



(A) 60 sq. cm

(B) 30 sq. cm

(C) 40 sq. cm

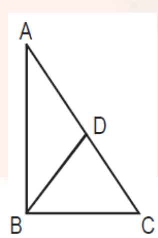
(D) 52 sq. cm

2 [B]

The radius of the circle is 6.5 cm. Therefore, its diameter = 13 cm and AB = 13 cm. Since the diameter of a circle subtends 90° at the circumference, $\angle\angle ACB = 90°$. So ΔACB is a right-angled triangle with AC = 5, AB = 13. Therefore, CB should be equal to 12 cm (as 5-12-13 form a Pythagorean triplet).

ACHIEVER SECTION

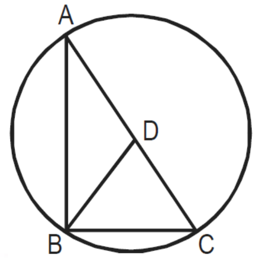
46. In ΔABC, $\angle B$ is a right angle, AC = 6 cm, and D is the mid-point of AC. The length of BD is



(A) 4 cm (B) $\sqrt{6}$ cm

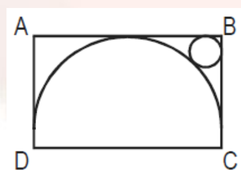
(C) 3 cm (D) 3.5 cm

Ans. [C]



In a right-angled triangle, the length median to the hypotenuse is half the length of the hypotenuse. Hence, $BD = \frac{1}{2}AC = 3\,cm$. This relationship can be verified by knowing that the diameter of a circle subtends a right angle at the circumference e.g. in the above figure D is the centre of the circle with AC as diameter. Hence, ∠∠ABC should be 90°. So BD should be the median to the hypotenuse. Thus, we can see that BD = AD = CD = Radius of this circle.

47. The figure shows the rectangle ABCD with a semicircle and a circle inscribed inside in it as shown. What is the ratio of the area of the circle to that of the semicircle?



(A) $\left(\sqrt{2}-1\right)^2 : 1$ (B) $2\left(\sqrt{2}-1\right)^2 : 1$

(C) $\left(\sqrt{2}-1\right)^2 : 2$ (D) None of these

Ans. [D]

48. A man travels three-fifths of a distance AB at a speed 3a, and the remaining at a speed 2b. If he goes from B to A and return at a speed 5c in the same time, then

(A) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ (B) a + b = c

(C) $\frac{1}{a}+\frac{1}{b}=\frac{2}{c}$ (D) None of these

Ans. [C]

Let the total distance be x. So the man travels a distance $\frac{3x}{5}$ at a speed 3a. Therefore, total time taken to travel this distance = $\frac{3x}{(15a)} = \frac{x}{(5a)}$

$\left[\text{time} = \frac{\text{distance}}{\text{speed}}\right]$

He then travels a distance $\frac{2x}{5}$ at a speed 2b. Hence,

time taken to travel this distance = $\frac{2x}{(10b)} = \frac{x}{(5b)}$. So

total time taken in going from A to B = $\frac{x}{(5a)} + \frac{x}{(5b)}$. Now he travels from B to A and comes back. So total distance travelled = 2x at an average speed 5c.

Hence, time taken to return = $\frac{2x}{(5c)}$.

Since the time taken in both the cases remains the same, we can write $\frac{x}{5a} + \frac{x}{5b} = \frac{2x}{5c}$.

Therefore, $\frac{1}{a} + \frac{1}{b} = \frac{2}{c}$

49. Three circles A, B and C have a common centre O. A is the inner circle, B middle circle and C is outer circle. The radius of the outer circle C, OP cuts the inner circle at X and middle circle at Y such that OX = XY = YP. The ratio of the area of the region between the inner and middle circles to the area of the region between the middle and outer circle is:

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$

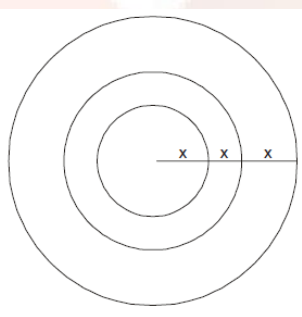
(C) $\frac{3}{5}$ (D) $\frac{1}{5}$

Ans. [C]

Area of circle = πr2

$\therefore$ Required ratio

$$= \frac{\pi(2x)^2 - \pi(x^2)}{\pi(3x)^2 - \pi(2x)^2} = \frac{\pi x^2(4-1)}{\pi x^2(9-4)} = \frac{3}{5}$$



50. The sides of a rhombus ABCD measure 2 cm each and the difference between two angles is 90° then the area of the rhombus is:

(A) √2 sq cm (B) 2√2 sq cm

(C) $3\sqrt{2}$ sq cm (D) $4\sqrt{2}$ sq cm

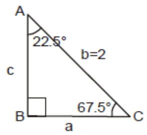
Ans. [B]

From the adjoining diagram,

$x + y = 90°$

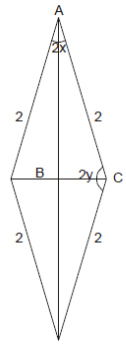
$x - y = 45°$

$x = 67.5°$ and $y = 22.5°$



Consider ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Thus, area of rhombus $= 2\sqrt{2}$ cm^2